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Investigating Supersymmetry Breaking

From Neutralino Mass Limits to Renormalization Group Invariants

PAAVO TIITOLA

University of Helsinki and Helsinki Institute of Physics,
P.O.Box 64 (Gustaf Hällströmin katu 2)
FIN-00014 University of Helsinki, Finland

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SUPERVISOR	Professor, Director Katri Huitu Helsinki Institute of Physics University of Helsinki Helsinki, Finland
PRE-EXAMINERS	Professor Tatsuo Kobayashi Department of Physics Hokkaido University Sapporo, Japan Professor Anindya Datta Department of Physics University of Calcutta Calcutta, India
OPPONENT	Associate Professor Emidio Gabrielli Department of Physics University of Trieste Trieste, Italy

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Paavo Tiitola
Helsinki, November 2019

Abstract

In this thesis I examine the currently most researched methods of supersymmetry (SUSY) breaking, a mechanism necessary to make the minimally supersymmetric extension to the Standard Model (MSSM) and its extensions consistent with the non-observation of the so-called superpartners of the Standard Model particles.

In the first part of the thesis I review the basic principles and features of SUSY, the MSSM, and SUSY breaking. In the remaining part I present the results of the three published articles that the thesis is based on.

In the first of our papers we used SUSY breaking mechanism dependent relations between gaugino mass parameters to estimate lower mass limits for neutralinos and charginos – few of the most promising candidates for near future experimental detection in SUSY. We then compared these limits in different SUSY breaking scenarios. We evaluated an upper bound on the mass of the lightest neutralino that follows from the structure of the mass matrix. We also examined cosmological implications of the SUSY breaking mechanism by calculating its effect on relic density. We studied the branching ratios of particle decay in each type of mechanism.

In the second paper we studied the effect of including a so-called dimension five operator in the MSSM on neutralino and chargino masses and composition, and examined the implications on determining the SUSY breaking mechanism. We also examined the usefulness of two sum rules in determining the SUSY breaking mechanism in this model.

In the final paper we examined quantities known as renormalization group invariants (RGIs) from the point of view of SUSY breaking. We discussed the potential role of these scale-invariant combinations of masses and couplings in determining the nature of SUSY breaking by solving the SUSY breaking parameters in terms of the RGIs for a general model of SUSY breaking, the so called deflected mirage mediation, which includes contributions from three main SUSY breaking mechanisms.

List of included articles

The research papers included in this thesis are:

[1] K. Huitu, J. Laamanen, P. N. Pandita and P. Tiitola,
Implications of different supersymmetry breaking patterns for the spectrum and decay of neutralinos and charginos
Phys. Rev. D **82** (2010) 115003
doi:10.1103/PhysRevD.82.115003 [arXiv:1006.0661 [hep-ph]].

[2] K. Huitu, P. N. Pandita and P. Tiitola,
Neutralino and chargino masses and related sum rules beyond MSSM
Phys. Lett. B **716** (2012) 298
doi:10.1016/j.physletb.2012.07.070 [arXiv:1103.4782 [hep-ph]].

[3] K. Huitu, P. N. Pandita and P. Tiitola,
Renormalization group invariants and sum rules in the deflected mirage mediation supersymmetry breaking
Phys. Rev. D **92** (2015) no.7 075037
doi:10.1103/PhysRevD.92.075037 [arXiv:1505.03455 [hep-ph]].

Author's contribution

For [1] I created the code necessary to extend SOFTSUSY to calculating mass spectrum for the mirage mediation scenario. I also performed the analysis on sum rules and wrote the corresponding text. I also wrote the text on deflected mirage mediation. I made the calculations and plotted the mass upper limits and created the table for lower mass limits. In [2] I performed the calculations, the plots, and wrote the corresponding analysis. The idea for the article, the introduction, parts of analysis and conclusion were by Katri and Pran. In [3] I performed the calculations, created the plots, and wrote the analysis. The original idea to study RGIs and deflected mirage mediation were by Katri and Pran. The idea to calculate the analytic expressions for the parameters in terms of RGIs was mine.

Contents

Chapter 1

Supersymmetric extensions to the Standard Model

While the Standard Model provides extremely accurate model to predict all currently available data on particles and their interactions it has several weaknesses and there is motivation to treat it as a low energy approximation of a more fundamental theory, from which it will be distinguished when experimental results at higher energy scales will become available. One property is the fine tuning required to bring the Higgs mass to the observed value which is unnaturally low compared to the expectations following from the radiative corrections received by the Higgs boson that are unconstrained by symmetry. This is commonly referred to as the hierarchy problem. Further reasons to be dissatisfied include the disappointing failure of the gauge coupling constants to completely unify at high energy scales, and the absence of a field that could explain the astronomical observations of high abundance of dark matter in the universe.

Supersymmetry is considered by many to be the leading idea on which to base an attempt to extend the Standard Model. For one it solves the hierarchy problem by cancelling the radiative corrections of the Higgs boson by introducing a supersymmetric partner to every field which results in a corresponding cancelling term in each order of the perturbation series. In the minimal supersymmetric extension of the Standard Model the gauge couplings unify with high precision as opposed to the slight deviation in the SM. Supersymmetric extensions also provide a dark matter candidate in the form of the lightest supersymmetric particle which does not decay and can fill the universe in a sufficient density to explain the extra mass observed.

Supersymmetry imposed as a local symmetry instead of global provides a theory of supergravity, thus creating a connection to general relativity. Also, supersymmetry is the only way to combine internal and spacetime symmetries that does not violate the Coleman-Mandula no-go theorem, and thus with supersymmetry all allowed space-time symmetries are included. For extensive introductions to supersymmetry, see [4], [5].

1.1 Supersymmetry algebra

In simple terms supersymmetry is a transformation that turns fermions into bosons and vice versa. In the language of Hilbert space formalism this can be written as

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle, \qquad Q|\text{Fermion}\rangle = |\text{Boson}\rangle, \tag{1.1.1}$$

where Q is an anti-commuting fermionic operator with spin angular momentum $\frac{1}{2}$ and the generator of supersymmetry. Since Q is fermionic, supersymmetry is a spacetime as well as internal symmetry. The ways in which the two types of symmetries can be combined is restricted by a theorem known as the Haag-Lopuszanski-Sohnius extension [6] of the Coleman-Mandula theorem [7]. It states that the form of such symmetries must satisfy

$$\{Q, Q^\dagger\} = P^\mu, \tag{1.1.2}$$
$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0, \tag{1.1.3}$$
$$[P^\mu, Q] = [P^\mu, Q^\dagger] = 0. \tag{1.1.4}$$

Supersymmetric gauge theories should thus be formulated with irreducible representations of the algebra (1.1.2) - (1.1.4) as particle states. Such representations are referred to as supermultiplets.

From the relation (1.1.4) it follows that the supersymmetry generators commute with the mass squared operator P^2, implying that components of a given supermultiplet have the same mass. If supersymmetry generators are required to commute with gauge generators, the supermultiplets contain only fields with the same gauge quantum numbers. It follows that the given supermultiplet can only contain one SM particle, thus the particle content must be twice that of SM. To formulate a physical supersymmetric gauge theory, one needs to construct a Lagrangian which is renormalizable, and invariant under both gauge and supersymmetry transformations.

A simple supersymmetric toy model known as the Wess-Zumino model includes only a massless scalar field and its fermionic superpartner [8].

$$S = \int d^4x \, \left(\mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}}\right), \tag{1.1.5}$$

$$\mathcal{L}_{\text{scalar}} = \partial^\mu \phi^* \partial_\mu \phi, \qquad \mathcal{L}_{\text{fermion}} = i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi. \tag{1.1.6}$$

Supersymmetric transformation should transform a scalar into a fermion and vice versa. The simplest possibility such transformation for the scalar field is

$$\delta\phi = \epsilon\psi, \qquad \delta\phi^* = \epsilon^\dagger \psi^\dagger, \tag{1.1.7}$$

where ϵ^α is an infinitesimal, anti-commuting, two-component Weyl fermion object of the dimension of $[\text{mass}]^{-1/2}$; it parameterizes the supersymmetry transformation.

For the purposes of this thesis supersymmetry is taken to be global, i.e. ϵ^α is a constant, satisfying $\partial_\mu \epsilon^\alpha = 0$. Promoting supersymmetry to a local symmetry is a possible path for formulating gauge theories of gravitation, referred to as supergravity.

The scalar part of the Lagrangian transforms as

$$\delta\mathcal{L}_{\text{scalar}} \;=\; \epsilon\partial^\mu\psi \, \partial_\mu\phi^* + \epsilon^\dagger \partial^\mu \psi^\dagger \, \partial_\mu \phi. \tag{1.1.8}$$

In order for the action to be invariant under the supersymmetry transformation, the transformation of the scalar part of the Lagrangian should be canceled by the transformation of the fermion part.

This is accomplished with a fermion transformation defined as

$$\delta\psi_\alpha \;=\; -i(\sigma^\mu \epsilon^\dagger)_\alpha \, \partial_\mu \phi, \qquad \delta\psi^\dagger_{\dot\alpha} \;=\; i(\epsilon\sigma^\mu)_{\dot\alpha} \, \partial_\mu \phi^*. \tag{1.1.9}$$

After simplification the fermion part of the Lagrangian now transforms as

$$\begin{aligned} \delta\mathcal{L}_{\text{fermion}} \;&=\; -\epsilon\partial^\mu\psi \, \partial_\mu\phi^* - \epsilon^\dagger\partial^\mu\psi^\dagger \, \partial_\mu\phi \\ &\quad -\partial_\mu\left(\epsilon\sigma^\nu\overline{\sigma}^\mu\psi \, \partial_\nu\phi^* - \epsilon\psi \, \partial^\mu\phi^* - \epsilon^\dagger\psi^\dagger \, \partial^\mu\phi\right). \end{aligned} \tag{1.1.10}$$

This cancels with $\delta\mathcal{L}_{\text{scalar}}$ up to a total derivative, thus

$$\delta S = \int d^4x \, \left(\delta\mathcal{L}_{\text{scalar}} + \delta\mathcal{L}_{\text{fermion}}\right) = 0. \tag{1.1.11}$$

The Wess-Zumino model illustrates how models invariant under supersymmetry can be formulated using the transformations (1.1.7) and (1.1.9). In order to formulate a physical theory complete with gauge and Yukawa interactions, the usual procedure of modifying the partial derivatives of the Lagrangian to gauge covariant derivatives is performed.

The bosonic and fermionic degrees of freedom can be conveniently represented as functions of superspace known as supefields, which include anti-commuting Grassman variable coordinates in addition to conventional Minkowski spatial coordinates, to account for the fermionic degrees of freedom. One superfield contains a complete supermultiplet. Two types of supermultiplets appear in SUSY models, chiral supermultiplets and gauge supermultiplets. Chiral supermultiplets consist of a two-component Weyl spinor and a complex scalar field. Gauge supermultiplets consist of a spin -1 gauge boson field and a spin $-\frac{1}{2}$ Majorana spinor (called the gaugino field), which is equals to its charge conjugate.

1.2 Minimal Supersymmetric Standard Model (MSSM)

Extending the SM to a supersymmetric model of quantum fields can be performed in innumerable ways by increasing the number of generators of supersymmetry transformations. The model containing smallest number of new fields and interactions thus providing the solution of least added complexity is known as the Minimal Supersymmetric Standard Model. It contains one supersymmetry generator (N=1). MSSM introduces 105 new free parameters to those of the Standard Model. MSSM has the advantage of providing solutions to the three major shortcomings of the Standard Model. It maintains naturalness through the cancellations of the Higgs boson radiative correction terms responsible of the hierarchy problem by introducing new terms deriving of the supersymmetric partners. Further, MSSM also exhibits an apparent unification of gauge couplings at two-loop level that is absent in the SM. Finally, it includes viable candidates for a Weakly Interacting Massive Particle (WIMP), i.e. a particle that can explain the presence of dark matter in the universe completely or partially. This follows from the property known as R-parity conservation, which is required to stabilise the proton, and prevents the lightest supersymmetric particle (often abbreviated as WIMP) from decaying. The following review of MSSM mostly follows the thorough discussion of [4].

Names		spin 0	spin 1/2	$SU(3)_c$, $SU(2)_L$, $U(1)_y$
squarks, quarks	Q	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$\mathbf{3}$, $\mathbf{2}$, 1/3
(× 3 families)	$\bar{u}$	$\tilde{\bar{u}}_L(\tilde{u}_R)$	$\bar{u}_L \sim (u_R)^c$	$\bar{\mathbf{3}}$, $\mathbf{1}$, -4/3
	$\bar{d}$	$\tilde{\bar{d}}_L(\tilde{d}_R)$	$\bar{d}_L \sim (d_R)^c$	$\bar{\mathbf{3}}$, $\mathbf{1}$, 2/3
sleptons, leptons	L	$(\tilde{\nu}_{eL}, \tilde{e}_L)$	(ν_{eL}, e_L)	$\mathbf{1}$, $\mathbf{2}$, -1
(× 3 families)	$\bar{e}$	$\tilde{\bar{e}}_L(\tilde{e}_R)$	$\bar{e}_L \sim (e_R)^c$	$\mathbf{1}$, $\mathbf{1}$, 2
higgs, Higgsinos	H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	$\mathbf{1}$, $\mathbf{2}$, 1
	H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$\mathbf{1}$, $\mathbf{2}$, -1

Table 1.1: Chiral supermultiplet fields in the MSSM [4].

Names	spin 1/2	spin 1	$SU(3)_c$, $SU(2)_L$, $U(1)_y$
gluinos, gluons	$\tilde{g}$	g	$\mathbf{8}$, $\mathbf{1}$, 0
winos, W bosons	$\widetilde{W}^\pm$, $\widetilde{W}^0$	$W^\pm$, W^0	$\mathbf{1}$, $\mathbf{3}$, 0
bino, B boson	$\tilde{B}$	B	$\mathbf{1}$, $\mathbf{1}$, 0

Table 1.2: Gauge supermultiplet fields in the MSSM [4].

1.3 The particle spectrum of MSSM

In addition to the SM particle content, MSSM contains their partners and an extended Higgs sector. There are four electronically neutral fermions, the neutralinos, which are mixtures of neutral wino and bino gauge fermions and neutral Higgsinos. Charginos are charged fermions composed of charged wino and charged Higgsinos. Squarks are scalar SUSY partners of the SM quarks. Similarly sleptons are the scalar partners of the leptons. Gluino is a Majorana fermionic supersymmetric partner of the gluon. The particles are arranged into 7 chiral supermultiplets and 3 gauge supermultiplets, that are listed in Tables 1.1. and 1.2 [4].

The non-gauge interactions in a supersymmetric gauge theory are defined by the superpotential W, which is a holomorphic function of complex variables. The MSSM is defined by the superpotential

$$W = y_u^{ij} \bar{u}_i Q_j \cdot H_u - y_d^{ij} \bar{d}_i Q_j \cdot H_d - y_e^{ij} \bar{e}_i L_j \cdot H_d + \mu H_u \cdot H_d. \tag{1.3.1}$$

The superpotential is related to the interaction Lagrangian density by the relation

$$\mathcal{L}_{\text{int}} = \left(-\frac{1}{2}W^{ij}\psi_i\psi_j + W^iF_i\right) + \text{c.c.}, \tag{1.3.2}$$

where

$$W^{ij} \quad = \frac{\delta^2}{\delta\phi_i\delta\phi_j}W, \tag{1.3.3}$$

$$W^i \qquad = \frac{\delta W}{\delta\phi_i}, \tag{1.3.4}$$

and F_i is an auxiliary field that can be eliminated using its equation of motion along with its complex conjugate. The full scalar potential also contains so-called D-term potential and can be written as

$$V(\phi,\phi^*) = W_i^*W^i + \frac{1}{2}\sum_a g_a^2(\phi^*T^a\phi)^2, \tag{1.3.5}$$

where where T_a is a representation matrix under the gauge group. The fields appearing in (1.3.1) are the chiral superfields indicated in Table 1.1. The MSSM superpotential (1.3.1) is the most general phenomenologically viable potential that respects the conservation of a symmetry known as R-parity. R-parity, which is multiplicatively conserved, is defined by

$$R = (-)^{3B+L+2s} \tag{1.3.6}$$

where s is the spin of the particle, B and L are Baryon and Lepton numbers, respectively. Renormalizable terms that violate R-parity exist, but are excluded from the superpotential. R-parity is incorporated to insure that the model is compatible with the non-observation of the proton decay. It is worth noting that for all the SM particles $P_R = +1$ while for the super partners $P_R = -1$. Thus a decay of supersymmetric particle into a SM particle is not allowed, and the superpartner with the lightest mass is stable.

1.3.1 Chargino and neutralino mass matrices

As a result of the electroweak symmetry breaking, non-diagonal terms are generated to the mass matrix of Higgsinos and eletroweak gauginos. As a result, the neutral Higgsinos ($\widetilde{H}_u^0$ and $\widetilde{H}_d^0$) and the neutral gauginos ($\widetilde{B}$, $\widetilde{W}^0$) mix, forming four mass eigenstates called neutralinos. Similarly, charged Higgsinos ($\widetilde{H}_u^+$ and $\widetilde{H}_d^-$) and winos ($\widetilde{W}^+$ and $\widetilde{W}^-$) combine to form two mass eigenstates with charge ±1 called charginos.

In the wino-Higgsino basis

$$\psi_j^+ \;=\; (-i\lambda^+, \psi_{H_d}^1), \quad \psi_j^- = (-i\lambda^-, \psi_{H_u}^2), \quad j = 1,\, 2, \tag{1.3.7}$$

where $\lambda^\pm = (1/\sqrt{2})(\lambda^1 \mp \lambda^2)$, and the superscripts 1, 2 refer to $SU(2)_L$ indices, the chargino mass matrix can be written as [9]

$$\mathcal{M}_\pm = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix}, \tag{1.3.8}$$

where M_2 is the supersymmetry breaking $SU(2)_L$ gaugino mass, μ is the Higgs(ino) mixing parameter, and $\tan\beta$ is the ratio v_u/v_d, where v_u, v_d are the vacuum expectation values of the neutral components of the two Higgs doublets H_d and H_u. We denote the eigenstates of the chargino mass matrix (1.3.8) as $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_2^\pm$, with eigenvalues $M_{\tilde{\chi}^\pm_{i=1,2}}$, respectively. The eigenvalues are most easily obtained from the diagonalization of ${\mathcal{M}_\pm}^\dagger \mathcal{M}_\pm$ resulting in the squares of the chargino masses

$$\begin{aligned} M^2_{\chi^\pm_{1,2}} =& \frac{1}{2}(M_2^2 + \mu^2 + 2m_W^2 \\ & \mp \sqrt{(M_2^2 + \mu^2 + 2m_W^2)^2 - 4(M_2\mu - m_W^2 \sin 2\beta)^2}). \end{aligned} \tag{1.3.9}$$

On the other, in the bino-wino-Higgsino basis

$$\psi_j^0 = (-i\lambda', \; -i\lambda^3, \; \psi_{H_u}^1, \; \psi_{H_d}^2), \quad j = 1, \; 2, \; 3, \; 4, \tag{1.3.10}$$

where λ' and λ^3 are the two-component gaugino states corresponding to the $U(1)_Y$ and the third component of the $SU(2)_L$ gauge groups, respectively, and $\psi_{H_u}^1, \psi_{H_d}^2$ are the two-component Higgsino states, the neutralino mass matrix can be written as [9]

$$\mathcal{M}_0 = \begin{pmatrix} M_1 & 0 & -M_Z\cos\beta\sin\theta_W & M_Z\sin\beta\sin\theta_W \\ 0 & M_2 & M_Z\cos\beta\cos\theta_W & -M_Z\sin\beta\cos\theta_W \\ -M_Z\cos\beta\sin\theta_W & M_Z\cos\beta\cos\theta_W & 0 & -\mu \\ M_Z\sin\beta\sin\theta_W & -M_Z\sin\beta\cos\theta_W & -\mu & 0 \end{pmatrix}, \tag{1.3.11}$$

where M_1 is the supersymmetry breaking $U(1)_Y$ gaugino mass, and g' and g are the gauge couplings associated with the $U(1)_Y$ and the $SU(2)_L$ gauge groups, respectively, with $\tan\theta_W = g'/g$, and $M_Z^2 = (g^2 + g'^2)(v_u^2 + v_d^2)/2$. The neutralino mass matrix can be diagonalized by a unitary transformation N

$$N^\dagger \mathcal{M}_0 N \;=\; \mathcal{M}_0^{\text{diagonal}}. \tag{1.3.12}$$

Assuming CP conservation, this transformation is an orthogonal transformation. the eigenstates of the neutralino mass matrix are denoted by $\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$ with eigenvalues $M_{\tilde{\chi}^0_{i=1,2,3,4}}$, labeled in order of increasing mass. Explicit expressions for these can be obtained, but these are not very illuminating. The neutralinos are mixtures of gauginos and Higgsinos

$$\tilde{\chi}_i^0 \;=\; N_{i1}\lambda' + N_{i2}\lambda^3 + N_{i3}\psi^1_{H_u} + N_{i4}\psi^2_{H_d}. \tag{1.3.13}$$

One can obtain information on the neutralino masses by studying the expansion of the neutralino mass matrix (1.3.11) in terms of M_Z/μ for $M_Z \ll \mu$. This expansion is obtained most conveniently by using the basis $(-i\tilde{\gamma}, -i\tilde{Z}^0, \tilde{H}_a^0, \tilde{H}_b^0)$, where

$$\tilde{\gamma} \;=\; \frac{1}{\sqrt{g^2+g'^2}}(g'\lambda^3 + g\lambda'), \tag{1.3.14}$$

$$\tilde{Z}^0 \;=\; \frac{1}{\sqrt{g^2+g'^2}}(g\lambda^3 - g'\lambda'), \tag{1.3.15}$$

$$\tilde{H}_a^0 \;=\; \frac{1}{\sqrt{v_1^2+v_2^2}}(v_1\psi^1_{H_u} - v_2\psi^2_{H_d}), \tag{1.3.16}$$

$$\tilde{H}_b^0 \;=\; \frac{1}{\sqrt{v_1^2+v_2^2}}(v_2\psi^1_{H_u} + v_1\psi^2_{H_d}), \tag{1.3.17}$$

are the photino, zino, and linear combinations of Higgsino states. In this basis, after a similarity transformation (see *e.g.* [10]), the neutralino mass matrix can be written as

$$\widetilde{\mathcal{M}_0} = \begin{pmatrix} M_1 & 0 & -M_Z\cos\left(\beta-\frac{\pi}{4}\right)s_W & M_Z\sin\left(\beta-\frac{\pi}{4}\right)s_W \\ 0 & M_2 & M_Z\cos\left(\beta-\frac{\pi}{4}\right)c_W & -M_Z\sin\left(\beta-\frac{\pi}{4}\right)c_W \\ -M_Z\cos\left(\beta-\frac{\pi}{4}\right)s_W & M_Z\cos\left(\beta-\frac{\pi}{4}\right)c_W & \mu & 0 \\ M_Z\sin\left(\beta-\frac{\pi}{4}\right)s_W & -M_Z\sin\left(\beta-\frac{\pi}{4}\right)c_W & 0 & -\mu \end{pmatrix}. \tag{1.3.18}$$

The mass matrix (1.3.18) can be diagonalized by using perturbation theory for values of $M_Z \ll \mu$. For the case $M_1 < M_2$, which is what one obtains in gravity mediated supersymmetry breaking (see below), the mass of the lightest neutralino can be written as, up to terms of $\mathcal{O}(M_Z/\mu)^2$,

$$m_{\chi_1^0} = M_1 - \frac{M_Z^2 s_W^2}{\mu}\sin 2\beta - \frac{1}{\mu^2}\left(M_Z^2 s_w^2 M_1 + \frac{M_Z^4 s_W^2 c_W^2}{M_2-M_1}\sin^2 2\beta\right). \tag{1.3.19}$$

Similarly, for the second lightest neutralino χ_2^0 one obtains

$$m_{\chi_2^0} = M_2 - \frac{M_Z^2 c_W^2}{\mu}\sin 2\beta - \frac{1}{\mu^2}\left(M_Z^2 c_W^2 M_2 + \frac{M_Z^4 s_W^2 c_W^2}{M_1-M_2}\sin^2 2\beta\right), \tag{1.3.20}$$

where $c_W^2 \equiv \cos^2\theta_W$ and $s_W^2 \equiv \sin^2\theta_W$. If instead $M_2 < M_1$, a situation that arises in anomaly mediated supersymmetry breaking models, Eq. (1.3.20) would represent the mass of the lightest neutralino χ_1^0, and Eq. (1.3.19) would give the formula for the mass of the second lightest neutralino. The dependence of the lightest neutralino mass on the specific SUSY breaking scenario is due to the fact that the ordering of the gaugino mass parameters is model dependent. Thus if $|\mu|$ value is small compared to $M_{1,2}$, Higgsino can form a large or even dominant component of the lightest neutralino, as can be seen from the mass formulae for the remaining two neutralinos:

$$m_{\tilde{\chi}_3^0} = \mu + \tfrac{M_Z^2}{2}(1+\sin 2\beta)\tfrac{\mu - s_W^2 M_2 - c_W^2 M_1}{(\mu-M_1)(\mu-M_2)} + \tfrac{M_Z^4}{8\mu^3}\cos^2 2\beta, \qquad (1.3.21)$$

and

$$m_{\tilde{\chi}_4^0} = -\mu - \tfrac{M_Z^2}{2}(1-\sin 2\beta)\tfrac{\mu + s_W^2 M_2 + c_W^2 M_1}{(\mu+M_1)(\mu+M_2)} - \tfrac{M_Z^4}{8\mu^3}\cos^2 2\beta. \qquad (1.3.22)$$

1.3.2 The Higgs sector

The Higgs sector is defined by the superpotential and the soft terms. The potential includes two complex Higgs doublets $H_u = (H_u^+,\ H_u^0)$ and $H_d = (H_d^0,\ H_d^-)$. This makes the Higgs sector more complicated than that of the SM, which has one complex scalar doublet.

The tree level scalar potential for the Higgses reads

$$\begin{aligned} V_{tree}(H_u, H_d) &= m_1^2|H_u|^2 + m_2^2|H_d|^2 - m_3^2(H_u H_d + h.c.) \\ &+ \frac{g^2+g'^2}{8}(|H_u|^2 - |H_d|^2)^2 + \frac{g^2}{2}|H_u H_d|^2, \end{aligned} \qquad (1.3.23)$$

where $m_1^2 = m_{H_u}^2 + \mu^2, m_2^2 = m_{H_d}^2 + \mu^2$. At the GUT scale $m_1^2 = m_2^2 = m_0^2 + \mu_0^2,\ m_3^2 = -B\mu_0$.

To find a mininum at non-zero the minimization condition is written as

$$\begin{aligned} \frac{1}{2}\frac{\delta V}{\delta H_u} &= m_1^2 v_1 - m_3^2 v_2 + \frac{g^2+g'^2}{4}(v_1^2 - v_2^2)v_1 = 0, && (1.3.24) \\ \frac{1}{2}\frac{\delta V}{\delta H_d} &= m_2^2 v_2 - m_3^2 v_1 + \frac{g^2+g'^2}{4}(v_1^2 - v_2^2)v_2 = 0, && (1.3.25) \end{aligned}$$

where the notation

$$< H_u >\equiv v_1 = v\cos\beta, \quad < H_d >\equiv v_2 = v\sin\beta, \qquad v^2 = v_1^2 + v_2^2, \quad \tan\beta \equiv \frac{v_2}{v_1}$$

is introduced. Solution of eqs.(1.3.24),(1.3.25) can be expressed in terms of v^2 and $\sin\beta$:

$$v^2 = \frac{4(m_1^2 - m_2^2\tan^2\beta)}{(g^2+g'^2)(\tan^2\beta - 1)}, \quad \sin 2\beta = \frac{2m_3^2}{m_1^2+m_2^2}. \tag{1.3.26}$$

Real positive solutions exist only if [11]:

$$m_1^2 + m_2^2 > 2m_3^2, \qquad m_1^2 m_2^2 < m_3^4, \tag{1.3.27}$$

which is not the case at the GUT scale.

This means that spontaneous breaking of the $SU(2)$ gauge invariance, which is needed in the SM to give masses for all the particles, does not take place in the MSSM at the tree level. However the condition 1.3.27 is satisfied at the electroweak scale after renormalization giving rise to a phenomenon known as the radiative symmetry breaking.

After electroweak symmetry is broken, three of the scalar degrees of freedom become the Nambu-Goldstone bosons G^0, $G^\pm$, which are then transformed to become the longitudinal modes of the Z^0 and $W^\pm$ massive vector bosons.

The procedure results in five Higgs mass eigenstate fields. The CP-even neutral scalars h^0 and H^0, of which h_0 is the lighter, CP-odd neutral scalar A^0, and the charged scalars H^+ and its conjugate charge scalar H^-.

The physical Higgs bosons acquire the following masses [12]:

$$\begin{aligned} \text{CP-odd neutral Higgs } A: &\qquad m_A^2 = m_1^2 + m_2^2, \\ \text{Charge Higgses } H^\pm: &\qquad m_{H^\pm}^2 = m_A^2 + M_W^2, \\ \text{CP-even neutral Higgses } H,h: & \end{aligned} \tag{1.3.28}$$

$$m_{H,h}^2 = \frac{1}{2}\left[m_A^2 + M_Z^2 \pm \sqrt{(m_A^2+M_Z^2)^2 - 4m_A^2 M_Z^2\cos^2 2\beta}\right], \tag{1.3.29}$$

where

$$M_W^2 = \frac{g^2}{2}v^2, \quad M_Z^2 = \frac{g^2+g'^2}{2}v^2,$$

and the mixing angle α is given by

$$\tan 2\alpha = -\tan 2\beta\left(\frac{m_A^2+M_Z^2}{m_A^2-M_Z^2}\right).$$

1.3.3 Higgs sector beyond the MSSM

From (1.3.29) we can infer that

$$m_h \leq M_Z |\cos 2\beta| \leq M_Z. \tag{1.3.30}$$

In other words, at the tree level, the mass of the lightest Higgs boson is bounded from above by the mass of the Z boson, and the observed Higgs mass violates this bound. However, there are large radiative corrections to the tree level bound [13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. The dominant radiative corrections to the Higgs mass come from the top-stop loops, and in order for these to be significant one of the stop mass eigenstates should be heavy. However, for the radiative corrections to account for the lightest Higgs boson mass, the top squarks must be so massive, that it makes the theory appear finely tuned. Alternatively, there must be large left-right mixing between scalar top quarks. While such large mixing is possible, it is rather difficult to obtain in specific models and can arise only from rather special points in the parameter space [23].

As pointed out by Dine, Seiberg and Thomas [24], this suggests that there are likely to be additional degrees of freedom in the theory beyond those of the MSSM.

There are several candidates for such additional physics beyond the MSSM [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40].

If this new physics lies at an energy scale M, which is above the masses of the MSSM degrees of freedom, one can study the effects of such additional degrees of freedom by using an effective Lagrangian from which the the physics at scale M has been integrated out.

In this effective field theory approach, which we review in more detail in Chapter 4, the effect of the high scale dynamics is contained in dimension five and higher operators in the Lagrangian that are suppressed by an approriate power of $1/M$. For a given observable only a small number of operators, with the smallest power of $1/M$ contribute.

It turns out that at dimension five only two operators are important for the Higgs sector [24]

$$W_5 \;=\; \mu H_u H_d + \frac{\lambda}{M}(H_u H_d)^2, \tag{1.3.31}$$

where M is an energy scale which is much above the typical masses of the MSSM fields, and λ is a dimensionless coupling. The dimension five operator in (1.3.31) raises the lightest Higgs boson mass above 125 GeV without fine tuning, and, hence, without loss of naturalness [24].

The superpotential (1.3.31) leads, up to fimension five, to the following interaction Lagrangian involving only the Higgsino ($\widetilde{H}_u$, $\widetilde{H}_d$) and the Higgs (H_u, H_d) fields [24]:

$$\mathcal{L} = \mu(\widetilde{H}_u\widetilde{H}_d)$$
$$-\frac{\epsilon_1}{\mu*}\left[2(H_uH_d)(\widetilde{H}_u\widetilde{H}_d) + 2(\widetilde{H}_uH_d)(H_u\widetilde{H}_d) + (H_u\widetilde{H}_d)(H_u\widetilde{H}_d) + (\widetilde{H}_uH_d)(\widetilde{H}_uH_d)\right]$$
$$+ \text{H.c.}, \quad (1.3.32)$$

where $SU(2)_L$ contraction between the fields in round parentheses is implied, and where

$$\epsilon_1 = \lambda\mu^*/M. \quad (1.3.33)$$

For definiteness, we take μ to be real in this thesis.

1.3.4 Renormalization group equations

In order to derive physical predictions at observable scales for the gauge couplings and masses from the input scale values of the parameters and vice versa, we need the information on how the masses and couplings evolve with respect to scale. the renormalization group equations are necessary. This is described by the renormalization group equations.

In both the Standard Model and MSSM the running at one-loop level for gauge couplings is determined by

$$\beta_{g_a} = \frac{d}{dt}g_a \;=\; \frac{1}{16\pi^2}b_a g_a^3, \quad (1.3.34)$$

where, for the SM the coefficients b_i are:

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_{Fam}\begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{Higgs}\begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}. \quad (1.3.35)$$

Here N_{Fam} is the number of generations of matter multiplets and N_{Higgs} is the number of Higgs doublets. $N_{Fam} = 3$ and $N_{Higgs} = 1$ is used for the SM.

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{Fam} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix}, \tag{1.3.36}$$

where we use $N_{Fam} = 3$ and $N_{Higgs} = 2$, values corresponding to the MSSM.

As we know, the Standard Model couplings do not completely unify at the GUT-scale, wheres the MSSM couplings unify satisfyingly with TeV-scale SUSY masses.

The renormalization of MSSM and other $N = 1$ supersymmetric theories is aided by the supersymmetric non-renormalization theorem which states that that the logarithmically divergent contributions to a particular process can always be written in terms of wave-function renormalizations, without any coupling vertex renormalization. The renormalization of softly broken SUSY gauge theories is described in detail e.g. [41]. A handy summary of the MSSM beta functions can be found in [143].

We assume here that soft supersymmetry breaking mass matrices are flavour diagonal and the first and second generation masses are degenerate. The trilinear couplings are assumed proportional to the Yukawa couplings: $\mathbf{a_u} = A_u \mathbf{y_u}$, $\mathbf{a_d} = A_d \mathbf{y_d}$, $\mathbf{a_e} = A_e \mathbf{y_e}$. The first and the second generation Yukawas are expected to be small, and are thus neglected from the analysis.

. For clarity we define

$$\beta(p) \equiv 16\pi^2 \frac{dp}{dt}. \tag{1.3.37}$$

Here p is a running parameter and $t \equiv \log(\mu/\mu_0)$, where μ is the renormalization scale and μ_0 an energy scale that makes the argument of the logarithm dimensionless. The following running parameters remain after the approximations:

$g_a \quad (a = 1, 2, 3)$	Gauge couplings
$M_a \quad (a = 1, 2, 3)$	Gaugino masses
$m^2_{\widetilde{Q}}, m^2_{\widetilde{\bar{u}}}, m^2_{\widetilde{\bar{d}}}, m^2_{\widetilde{L}}, m^2_{\widetilde{\bar{e}}}$	Fermion masses
$m^2_{H_u}, m^2_{H_d}$	Higgs mass parameters
y_t, y_b, y_τ	Yukawa couplings for the third generation (s)fermions
A_t, A_b, A_τ	Trilinear couplings for the third generation sfermions
μ	Supersymmetry respecting Higgs mixing parameter

Here we use the soft Higgs mixing parameter $B = b/\mu$ rather than b because its β-function is simpler. For the sfermion masses, we denote the first, second, and third generation with a subscripts 1, 2, and 3.

For convenience we define

$$\begin{aligned} D_Y &\equiv \mathrm{Tr}\left(Y m^2\right) \\ &= \sum_{\text{gen}} \left(m^2_{\tilde{Q}} - 2m^2_{\tilde{\bar{u}}} + m^2_{\tilde{\bar{d}}} - m^2_{\tilde{L}} + m^2_{\tilde{\bar{e}}}\right) + m^2_{H_u} - m^2_{H_d}. \end{aligned} \tag{1.3.38}$$

Here the trace runs over all chiral multiplets and the sum runs over the three sfermion generations. Furthermore, we define the useful combinations:

$$X_t = 2|y_t|^2 \left(m^2_{H_u} + m^2_{\tilde{Q}_3} + m^2_{\tilde{\bar{u}}_3} + |A_t|^2\right) \tag{1.3.38a}$$

$$X_b = 2|y_b|^2 \left(m^2_{H_d} + m^2_{\tilde{Q}_3} + m^2_{\tilde{\bar{d}}_3} + |A_b|^2\right) \tag{1.3.38b}$$

$$X_\tau = 2|y_\tau|^2 \left(m^2_{H_d} + m^2_{\tilde{L}_3} + m^2_{\tilde{\bar{e}}_3} + |A_\tau|^2\right) \tag{1.3.38c}$$

Then the resulting β-functions for the MSSM are:

$$\beta(g_a) = b_a g_a^3 \qquad (a = 1, 2, 3) \tag{1.3.38d}$$

$$\beta(M_a) = 2b_a g_a^2 M_a \qquad (a = 1, 2, 3) \tag{1.3.38e}$$

$$\beta(m^2_{\tilde{Q}_{1,2}}) = -\frac{2}{15} g_1^2 M_1^2 - 6 g_2^2 M_2^2 - \frac{32}{3} g_3^2 M_3^2 + \frac{1}{5} g_1^2 D_Y \tag{1.3.38f}$$

$$\beta(m^2_{\tilde{\bar{u}}_{1,2}}) = -\frac{32}{15} g_1^2 M_1^2 - \frac{32}{3} g_3^2 M_3^2 - \frac{4}{5} g_1^2 D_Y \tag{1.3.38g}$$

$$\beta(m^2_{\tilde{\bar{d}}_{1,2}}) = -\frac{8}{15} g_1^2 M_1^2 - \frac{32}{3} g_3^2 M_3^2 + \frac{2}{5} g_1^2 D_Y \tag{1.3.38h}$$

$$\beta(m^2_{\tilde{L}_{1,2}}) = -\frac{6}{5} g_1^2 M_1^2 - 6 g_2^2 M_2^2 - \frac{3}{5} g_1^2 D_Y \tag{1.3.38i}$$

$$\beta(m^2_{\tilde{\bar{e}}_{1,2}}) = -\frac{24}{5} g_1^2 M_1^2 + \frac{6}{5} g_1^2 D_Y \tag{1.3.38j}$$

$$\beta(m^2_{\tilde{Q}_3}) = X_t + X_b - \frac{2}{15} g_1^2 M_1^2 - 6 g_2^2 M_2^2 - \frac{32}{3} g_3^2 M_3^2 + \frac{1}{5} g_1^2 D_Y \tag{1.3.38k}$$

$$\beta(m^2_{\tilde{\bar{u}}_3}) = 2X_t - \frac{32}{15} g_1^2 M_1^2 - \frac{32}{3} g_3^2 M_3^2 - \frac{4}{5} g_1^2 D_Y \tag{1.3.38l}$$

$$\beta(m^2_{\tilde{\bar{d}}_3}) = 2X_b - \frac{8}{15} g_1^2 M_1^2 - \frac{32}{3} g_3^2 M_3^2 + \frac{2}{5} g_1^2 D_Y \tag{1.3.38m}$$

$$\beta(m^2_{\tilde{L}_3}) = X_\tau - \frac{6}{5} g_1^2 M_1^2 - 6 g_2^2 M_2^2 - \frac{3}{5} g_1^2 D_Y \tag{1.3.38n}$$

$$\beta(m^2_{\tilde{\bar{e}}_3}) = 2X_\tau - \frac{24}{5}g_1^2M_1^2 + \frac{6}{5}g_1^2D_Y \tag{1.3.38o}$$

$$\beta(m^2_{H_u}) = 3X_t - \frac{6}{5}g_1^2M_1^2 - 6g_2^2M_2^2 + \frac{3}{5}g_1^2D_Y \tag{1.3.38p}$$

$$\beta(m^2_{H_d}) = 3X_b + X_\tau - \frac{6}{5}g_1^2M_1^2 - 6g_2^2M_2^2 - \frac{3}{5}g_1^2D_Y \tag{1.3.38q}$$

$$\beta(y_t) = y_t\left[6|y_t|^2 + |y_b|^2 - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\right] \tag{1.3.38r}$$

$$\beta(y_b) = y_b\left[6|y_b|^2 + |y_t|^2 + |y_\tau|^2 - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\right] \tag{1.3.38s}$$

$$\beta(y_\tau) = y_\tau\left[4|y_\tau|^2 + 3|y_b|^2 - \frac{9}{5}g_1^2 - 3g_2^2\right] \tag{1.3.38t}$$

$$\beta(\mu) = \mu\left[3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - \frac{3}{5}g_1^2 - 3g_2^2\right] \tag{1.3.38u}$$

$$\beta(A_t) = 12A_t|y_t|^2 + 2A_b|y_b|^2 + \frac{26}{15}g_1^2M_1 + 6g_2^2M_2 + \frac{32}{3}g_3^2M_3 \tag{1.3.38v}$$

$$\beta(A_b) = 12A_b|y_b|^2 + 2A_t|y_t|^2 + 2A_\tau|y_\tau|^2 + \frac{14}{15}g_1^2M_1 + 6g_2^2M_2 + \frac{32}{3}g_3^2M_3 \tag{1.3.38w}$$

$$\beta(A_\tau) = 8A_\tau|y_\tau|^2 + 6A_b|y_b|^2 + \frac{18}{5}g_1^2M_1 + 6g_2^2M_2 \tag{1.3.38x}$$

$$\beta(B) = 6A_t|y_t|^2 + 6A_b|y_b|^2 + 2A_\tau|y_\tau|^2 + \frac{6}{5}g_1^2M_1 + 6g_2^2M_2 \tag{1.3.38y}$$

1.3.5 Anomalous dimensions

Here we list the expressions for the anomalous dimensions that are required for computing the input scale masses. The anomalous dimensions γ^i_j at one-loop order are given by [4]:

In general, at 1-loop order,

$$\gamma^i_j = \frac{1}{16\pi^2}\left[\frac{1}{2}y^{imn}y^*_{jmn} - 2g_a^2C_a(i)\delta^i_j\right], \tag{1.3.39}$$

where $C_a(i)$ are the quadratic Casimir group theory invariants for the superfield Φ_i, defined in terms of the Lie algebra generators T^a by

$$(T^aT^a)_i{}^j = C_a(i)\delta_i^j \tag{1.3.40}$$

with gauge couplings g_a. Explicitly, for the MSSM supermultiplets:

$$C_3(i) = \begin{cases} 4/3 & \text{for } \Phi_i = Q, \overline{u}, \overline{d}, \\ 0 & \text{for } \Phi_i = L, \overline{e}, H_u, H_d, \end{cases} \tag{1.3.41}$$

$$C_2(i) = \begin{cases} 3/4 & \text{for } \Phi_i = Q, L, H_u, H_d, \\ 0 & \text{for } \Phi_i = \overline{u}, \overline{d}, \overline{e}, \end{cases} \tag{1.3.42}$$

$$C_1(i) = 3Y_i^2/5 \quad \text{for each } \Phi_i \text{ with weak hypercharge } Y_i. \tag{1.3.43}$$

We assume that only the Yukawa couplings of the third generation are significant. Then the anomalous dimensions become at one-loop order:

$$16\pi^2 \gamma_{H_u} = 3|y_t|^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2 \tag{1.3.43a}$$

$$16\pi^2 \gamma_{H_d} = 3|y_b|^2 + |y_\tau|^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2 \tag{1.3.43b}$$

$$16\pi^2 \gamma_{\widetilde{Q}_i} = \delta_{i3}\left(|y_t|^2 + |y_b|^2\right) - \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{1}{30}g_1^2 \tag{1.3.43c}$$

$$16\pi^2 \gamma_{\widetilde{\overline{u}}_i} = \delta_{i3} \cdot 2|y_t|^2 - \frac{8}{3}g_3^2 - \frac{8}{15}g_1^2 \tag{1.3.43d}$$

$$16\pi^2 \gamma_{\widetilde{\overline{d}}_i} = \delta_{i3} \cdot 2|y_b|^2 - \frac{8}{3}g_3^2 - \frac{2}{15}g_1^2 \tag{1.3.43e}$$

$$16\pi^2 \gamma_{\widetilde{L}_i} = \delta_{i3} \cdot |y_\tau|^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2 \tag{1.3.43f}$$

$$16\pi^2 \gamma_{\widetilde{\overline{e}}_i} = \delta_{i3} \cdot 2|y_\tau|^2 - \frac{6}{5}g_1^2 \tag{1.3.43g}$$

We will also need derivatives of the anomalous dimensions with respect to $t = \ln(\mu/\mu_0)$. These are given by:

$$(16\pi^2)^2 \dot{\gamma}_{H_u} = 6|y_t|^2 B_t - 3g_2^4 - \frac{99}{25}g_1^4 \tag{1.3.43h}$$

$$(16\pi^2)^2 \dot{\gamma}_{H_d} = 6|y_b|^2 B_b + 2|y_\tau|^2 B_\tau - 3g_2^4 - \frac{99}{25}g_1^4 \tag{1.3.43i}$$

$$(16\pi^2)^2 \dot{\gamma}_{\widetilde{Q}_i} = \delta_{i3}\left(2|y_t|^2 B_t + 2|y_b|^2 B_b\right) + 16g_3^4 - 3g_2^4 - \frac{11}{25}g_1^4 \tag{1.3.43j}$$

$$(16\pi^2)^2 \dot{\gamma}_{\widetilde{\overline{u}}_i} = \delta_{i3} \cdot 4|y_t|^2 B_t + 16g_3^4 - \frac{176}{25}g_1^4 \tag{1.3.43k}$$

$$(16\pi^2)^2 \dot{\gamma}_{\widetilde{\overline{d}}_i} = \delta_{i3} \cdot 4|y_b|^2 B_b + 16g_3^4 - \frac{44}{25}g_1^4 \tag{1.3.43l}$$

$$(16\pi^2)^2 \dot{\gamma}_{\widetilde{L}_i} = \delta_{i3} \cdot 2|y_\tau|^2 B_\tau - 3g_2^4 - \frac{99}{25}g_1^4 \tag{1.3.43m}$$

$$(16\pi^2)^2\dot{\gamma}_{\widetilde{\bar{e}}_i} = \delta_{i3} \cdot 4|y_\tau|^2 B_\tau - \frac{396}{25} g_1^4 \tag{1.3.43n}$$

where we have defined the following quantities for convenience:

$$B_t \equiv 6|y_t|^2 + |y_b|^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{15} g_1^2 \tag{1.3.43o}$$

$$B_b \equiv 6|y_b|^2 + |y_t|^2 + |y_\tau|^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{7}{15} g_1^2 \tag{1.3.43p}$$

$$B_\tau \equiv 4|y_\tau|^2 + 3|y_b|^2 - 3g_2^2 - \frac{9}{5} g_1^2 \tag{1.3.43q}$$

1.4 The lightest supersymmetric particle and dark matter

As we discussed, in models with R-parity conservation, such as the MSSM, each particle must eventually decay in to a state including the supersymmetric particle with the lightest mass. This field, abbreviated as LSP (Lightest Supersymmetric Particle) is stable as the R-parity prevents decay into an SM particle. Although R-parity violation would not render the theory inconsistent, the non-observation of proton decay makes the assumption well-motivated.

Presence of dark matter in significant quantities is by now a well established fact deduced from astronomical observations of galactic rotation curves, which imply that the galactic masses far exceed the portion explained by its content of luminous objects [42]. The most promising explanation for dark matter is a non-baryonic massive weakly interacting particle (abbreviated as WIMP) [43]. An electrically neutral (and thus unable to scatter electromagnetic radiation) LSP, such as the graviton or the neutralino would be a natural candidate for WIMP. Since all sparticles created in the Big Bang would have to have decayed into LSPs and LSPs are stable over cosmological timescales, the LSPs should permeate the universe and tend to propagate towards gravitational attractors such as galaxies, thus affecting the velocities of luminous objects within the galaxy. The relic density of the WIMPS can be calculated assuming that the WIMPs were in thermal and chemical equilibrium with SM particles after inflation. As WIMPs drop out of thermal equilibrium, co-moving WIMP density remains constant. The present relic density Ω_χ, can then be calculated from the present cosmic microwave background temperature:

$$\Omega_\chi h^2 \simeq \frac{T_0^3}{\mathcal{M}_{Pl}^3 < \sigma_a v >}, \tag{1.4.1}$$

where T_0 is the current cosmic microwave background temperature, σ_a is the total annihilation cross section of the WIMPs, and v is the relative velocity of two WIMPs in their centre of mass frame.

In Chapter 3 we examine various limits on the relic density in different supersymmetry breaking scenarios in which neutralino takes the part of the LSP.

Chapter 2

Supersymmetry breaking

2.1 Spontaneous supersymmetry breaking

The prediction of the existence of supersymmetric particles with mass similar to those of their Standard Model partners is incompatible with observational data, since no supersymmetric partners have been detected so far. Consequently, SUSY cannot be part of Nature unless it is broken at at the energy scales in which observations are currently performed. The breaking of supersymmetry should be incorporated to a SUSY model without reintroducing ultraviolet divergencies, in order not to lose one of the main assets of SUSY. This is achieved through soft supersymmetry breaking [44].

A way to produce soft supersymmetry breaking terms in the Lagrangian of the theory is the mechanism of spontaneous supersymmetry breaking. In spontaneous symmetry breaking as opposed to explicit symmetry breaking the vacuum is non-invariant under the symmetry while Lagrangian remains invariant. (A review of spontaneous supersymmetry breaking can be found in e.g. [4].) In the case of supersymmetry this means that the vacuum state $|0>$ is not annihilated by all the supersymmetry generators, that is $Q_\alpha|0>\neq 0$ and $Q_\alpha^\dagger|0>\neq 0$. From (1.1.2) it follows that the Hamiltonian can be written in terms of the supersymmetry generators as

$$H = P^0 = \frac{1}{4}(Q_1Q_1^\dagger + Q_1^\dagger Q_1 + Q_2Q_2^\dagger + Q_2^\dagger Q_2). \tag{2.1.1}$$

If supersymmetry is unbroken in the vacuum state, it follows that $H|0\rangle = 0$ and the vacuum has zero energy. On the other hand, if supersymmetry is spontaneously broken in the vacuum state, it follows the vacuum has positive energy.

In most cases $\langle 0|H|0\rangle = \langle 0|V|0\rangle$, where V is the scalar potential

$$V(\phi, \phi^*) = F^{*i}F_i + \frac{1}{2}\sum_a D^a D^a, \tag{2.1.2}$$

where F_i, and D^a are F- and D-components of some chiral supermultiplet.

We see that the vacuum has positive energy if the expectation value of either F_i or D has non-zero vacuum expectation value.

Supersymmetry breaking with a non-zero D-term VEV is known as the Fayet-Iliopoulos mechanism [45, 46] whereas models with non-zero F are referred to as O'Raifeartaigh models [47]. Fayet-Iliopoulos breaking mechanism suffers from difficulties in producing phenomenologically viable masses to all of the MSSM particles, whereas O'Raifeartaigh models are phenomenologically more acceptable. The models considered in this thesis are based on the O'Raifeartaigh mechanism of SUSY breaking. MSSM does not contain a an appropriate field with an F-term that could develop a VEV and therefore it is expected that the supersymmetry is broken spontaneously in a sector different from the MSSM called a hidden sector. The supersymmetry breaking is then communicated to the MSSM sector by some interaction.

The most general form of soft terms for the MSSM can be written as [4]:

$$\begin{aligned}
\mathcal{L}_{\text{soft}}^{\text{MSSM}} &= -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + \text{c.c.}\right) \\
&\quad - \left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d + \text{c.c.}\right) \\
&\quad -\widetilde{Q}^\dagger\,\mathbf{m^2_Q}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m^2_L}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m^2_{\overline{u}}}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m^2_{\overline{d}}}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m^2_{\overline{e}}}\,\widetilde{\overline{e}}^\dagger \\
&\quad - m^2_{H_u}H_u^*H_u - m^2_{H_d}H_d^*H_d - (bH_uH_d + \text{c.c.})\,,
\end{aligned} \tag{2.1.3}$$

where M_3, M_2, and M_1 are the gluino, wino, and bino mass terms. The second line in (2.1.3) contains the trilinear couplings a^{ijk}. Each of $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$ is a complex 3×3 matrix. The third line consists of squark and slepton mass terms $(m^2)^j_i$. Each of $\mathbf{m^2_Q}$, $\mathbf{m^2_{\overline{u}}}$, $\mathbf{m^2_{\overline{d}}}$, $\mathbf{m^2_L}$, $\mathbf{m^2_{\overline{e}}}$ is a 3×3 matrix in family space. The last line contains supersymmetry-breaking contributions to the Higgs potential. There are several candidates for the interaction that mediate supersymmetry breaking to the MSSM sector, leading to different supersymmetry breaking scenarios and different values for the parameters in (2.1.3).

The specific scenario is important in determining the masses of the supersymmetric particles and hence, the experimental signatures of supersymmetry. One of the ways to experimentally classify the different scenarios are the ratios of the gaugino masses. These mass patterns can then be used to derive limits to the mass of the LSP, or specifically in this thesis, the lightest neutralino.

2.2 Supersymmetry breaking models

2.2.1 Gravity mediated supersymmetry breaking

The model of supersymmetry breaking that has been studied most extensively is the gravity mediated [48, 49, 25, 50, 51, 52, 53] supersymmetry breaking model. In this class of models, supersymmetry is assumed to be broken in a hidden sector by fields which interact with the MSSM particles through only the gravitational interactions. Gravity can be included in a supersymmetric theory by making supersymmetry a local instead of a global symmetry. Spontaneous breaking of global supersymmetry implies existence of a massles Weyl fermion, known as the Goldstino, analoguous to the Goldstone boson involved in the electroweak symmetry breaking. When supersymmetry is promoted to a local symmetry and broken spontaneously, the spin 3/2 supersymmetric partner of the graviton, the gravitino, aqcuires mass by absorbing the Goldstino.

In the gravity mediated supersymmetry breaking scenarios, gravity is taken to be the interaction that mediates the breaking between the hidden and MSSM sectors. This means that the supergravity effective Lagrangian contains non-renormalizable terms that communicate between the two sectors and are suppressed by powers of the Planck mass $M_{\rm P}$. The form of these terms is determined by the underlying theory, but can be simplified by additional assumptions of universal gaugino and sfermion masses and trilinear couplings proportional to the Yukawa couplings.

Then the soft terms in $\mathcal{L}_{\rm soft}^{\rm MSSM}$ are all determined by just four parameters:

$$m_{1/2} = f\frac{\langle F\rangle}{M_{\rm P}}, \qquad m_0^2 = k\frac{|\langle F\rangle|^2}{M_{\rm P}^2}, \qquad A_0 = \alpha\frac{\langle F\rangle}{M_{\rm P}}, \qquad B_0 = \beta\frac{\langle F\rangle}{M_{\rm P}}. \quad (2.2.1)$$

In terms of (2.1.3) we have

$$M_3 = M_2 = M_1 = m_{1/2}, \quad (2.2.2)$$
$$\mathbf{m^2_Q} = \mathbf{m^2_{\bar{u}}} = \mathbf{m^2_{\bar{d}}} = \mathbf{m^2_L} = \mathbf{m^2_{\bar{e}}} = m_0^2\,\mathbf{1}, \qquad m^2_{H_u} = m^2_{H_d} = m_0^2, \quad (2.2.3)$$
$$\mathbf{a_u} = A_0\mathbf{y_u}, \qquad \mathbf{a_d} = A_0\mathbf{y_d}, \qquad \mathbf{a_e} = A_0\mathbf{y_e}, \quad (2.2.4)$$
$$b = B_0\mu, \quad (2.2.5)$$

at a renormalization scale $Q \approx M_{\mathrm{P}}$. This model is commonly referred to as mSUGRA. The hidden sector is usually assumed to decouple at the Planck scale and not to have light fields. The low energy masses are determined by evolving the above matching scale masses to the electroweak scale through the renormalization group equations. From (1.3.38a) and (1.3.38b) can be read that the soft gaugino masses M_i and the gauge couplings g_i satisfy the renormalization group equations

$$16\pi^2 \frac{dM_i}{dt} = 2b_i M_i g_i^2, \qquad b_i = \left(\frac{33}{5}, 1, -3\right), \tag{2.2.6}$$

$$16\pi^2 \frac{dg_i}{dt} = b_i g_i^3 \tag{2.2.7}$$

at the leading order, where $i = 1, 2, 3$ refer to the $U(1)_Y, SU(2)_L$ and the $SU(3)$ gauge groups, respectively. Furthermore, $g_1 = \frac{5}{3}g'$, $g_2 = g$, and g_3 is the $SU(3)_C$ gauge coupling.

2.2.2 Gauge mediated supersymmetry breaking

As the name suggests, in gauge mediated supersymmetry breaking scenario [54, 55, 56] the supersymmetry breaking is communicated from the hidden sector by gauge interactions. Gravity interaction is still present but taken to be negligible in magnitude compared to the gauge mediated effect. This is achieved by introducing new chiral supermultiplets that couple to the supersymmetry breaking source as well as to the matter fields through the gauge boson and gaugino interactions. In the limit of vanishing gauge couplings, the new multiplets decouple and MSSM sector and the hidden supersymmetry sector are separated. The effects of the hidden sector on the MSSM fields can be parametrised by the effective Lagrangian with the following mass terms:

$$M_a = g_a^2 B_a \qquad (a = 1, 2, 3) \tag{2.2.8}$$

$$m_i^2 = g_1^2 Y_i \zeta + \sum_{a=1}^{3} g_a^4 C_a(i) A_a, \tag{2.2.9}$$

where Y_i is the hypercharge of the scalar field Φ_i and $C_a(i)$ is the quadratic Casimir of the representation of Φ_i under the gauge group labeled by a.

The parameters ζ, A_a, B_a encode properties of the hidden sector in the most general parametrisation of gauge mediation known as the general gauge mediation [57, 58].

Usually it is assumed that the messengers to consist of N copies of the $\mathbf{5}+\overline{\mathbf{5}}$ of $SU(5)$ (which contains the SM gauge group). In this case (2.2.8) and (2.2.9) simplify to

$$
\begin{aligned}
M_a &= g_a^2 \Lambda N, && (2.2.10)\\
m_{\phi_i}^2 &= 2\Lambda^2 N \sum_{a=1}^{3} C_a(i) g_a^4, && (2.2.11)
\end{aligned}
$$

where Λ is the mass scale associated with the messengers.

2.2.3 Anomaly mediated supersymmetry breaking

It has been shown that in a general hidden sector model with supersymmetry breaking F-terms, soft mass terms are generally generated as a consequence of the breakdown of the superconformal invariance at the quantum level, known as the super-Weyl anomaly [59].

In some scenarios, for example, if the visible sector is confined to a (3+1)-brane and the supersymmetry breaking occurs on another brane or a compatified dimension, the tree level mass terms arising from e.g. gravity mediation are heavily suppressed. In this type of scenario the mass contributions generated via the anomaly can dominate. This mechanism of supersymmetry breaking is referred to as anomaly mediated supersymmetry breaking (AMSB) [59, 60].

The anomaly mediated contributions to the soft supersymmetry breaking parameters M_λ (gaugino mass), m_i^2 (soft scalar mass squared), and A_y (the trilinear supersymmetry breaking coupling, where y refers to the Yukawa coupling) can be written as

$$
\begin{aligned}
M_\lambda &= \frac{\beta_g}{g} m_{3/2}, && (2.2.12)\\
m_i^2 &= -\frac{1}{4}\left(\frac{\partial \gamma_i}{\partial g}\beta_g + \frac{\partial \gamma_i}{\partial y}\beta_y\right) m_{3/2}^2, && (2.2.13)\\
A_y &= -\frac{\beta_y}{y} m_{3/2}, && (2.2.14)
\end{aligned}
$$

where $m_{3/2}$ is the gravitino mass, β's are the relevant β functions, and γ's are the anomalous dimensions of the corresponding chiral superfields 1.3.39. Since the terms are proportional to their corresponding beta functions it follows that they are not to be understood only as boundary conditions at some matching scale but as renormalization group equations, valid at all energies.

An immediate consequence of these relations is that supersymmetry breaking terms are completely insensitive to the physics in ultraviolet. The anomalous dimensions and beta functions are completely determined by their values at a given energy, thus completely specifying the soft supersymmetry breaking parameters at all energies.

A major problem with the model is that the pure scalar mass-squared anomaly contribution for sleptons is negative [59]. There are a number of proposals for resolving this problem of tachyonic slepton masses [61, 62, 63, 64, 65, 66], but some of the solutions may spoil the most attractive feature of the anomaly mediated models, *i.e.*, the renormalization group (RG) invariance of the soft terms and the consequent ultraviolet insensitivity of the mass spectrum. A simple phenomenologically attractive way of parametrizing the nonanomaly mediated contributions to the slepton masses, so as to cure their tachyonic spectrum, is to add a common mass parameter m_0 to all the squared scalar masses [67]. However, a nonanomaly mediated term destroys the attractive feature of the RG invariance of soft masses.

The renormalization group evolution of the resulting model, nevertheless, inherits some of the simplicity of the pure anomaly mediated relations.

There are several alternative ways to generate these extra contributions to the soft squared masses in the anomaly mediated supersymmetry breaking scheme. In particular, there are models of supersymmetry breaking mediated through a small extra dimension, where SM matter multiplets and a supersymmetry breaking hidden sector are confined to opposite four-dimensional boundaries while gauge multiplets lie in the bulk. In this scenario the soft gaugino mass terms are due to the anomaly mediated supersymmetry breaking and therefore are governed by (2.2.12). On the other hand, scalar masses get contributions from both anomaly mediation and a tiny hard breaking of supersymmetry by operators on the hidden sector boundary. These operators contribute to scalar masses at one loop and this contribution is dominant, thereby making all squared scalar masses positive. The gaugino spectrum is unaltered, and the model resembles an anomaly mediated supersymmetry breaking model with nonuniversal scalar masses [68].

2.2.4 Mirage mediated supersymmetry breaking

In some cases the anomaly mediated contributions and mSUGRA type contributions to the soft masses can manifest at comparable values, leading to a scenario known as the mirage mediation. Such mass spectrum naturally arises from KKLT-type moduli stabilization in type IIb string theory, where modulus which determines the SM gauge couplings is stabilized by non-perturbative effects and SUSY is broken by a brane-localized source which is sequestered from the visible sector [150].

The soft masses of visible fields are determined by the modulus mediation - a mediation scenario which produces contributions of the mSUGRA form - and the anomaly mediation which are comparable to each other if the gravitino mass $m_{3/2} \sim 10$ TeV as required to give the weak scale size of soft masses.

Phenomenology and cosmology of mirage mediation have been studied in [70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81]. Signatures of the scenario at LHC and the spectrum of neutralino mass in particular have been studied in [82, 83].

The boundary conditions for the soft supersymmetry breaking terms at the GUT scale are as follows [84, 85]:

$$M_a = M_0\Big[\,1 + \frac{\ln(M_{Pl}/m_{3/2})}{16\pi^2} b_a g_a^2 \alpha\,\Big], \tag{2.2.15}$$

$$A_{ijk} = M_0\Big[\,(a_i + a_j + a_k) - \frac{\ln(M_{Pl}/m_{3/2})}{16\pi^2}(\gamma_i + \gamma_j + \gamma_k)\alpha\,\Big], \tag{2.2.16}$$

$$m_i^2 = M_0^2\Big[\,c_i - \frac{\ln(M_{Pl}/m_{3/2})}{16\pi^2}\theta_i \alpha - \left(\frac{\ln(M_{Pl}/m_{3/2})}{16\pi^2}\right)^2 \dot{\gamma}_i \alpha^2\,\Big], \tag{2.2.17}$$

where $\dot{\gamma}$, and θ_i are defined as

$$\dot{\gamma}_i = 2\sum_a g_a^4 b_a c_a(\Phi_i) - \sum_{lm} |y_{ilm}|^2 b_{y_{ilm}}, \tag{2.2.18}$$

$$\theta_i = 4\sum_a g_a^2 c_a(\Psi_i) - \sum_{j,k} |y_{ijk}|^2 (p - n_i - n_j - n_k), \tag{2.2.19}$$

in which $b_{y_{ilm}}$ is the beta function for the Yukawa coupling y_{ilm}, c_a is the quadratic Casimir operator for the field Ψ_i. $M_0 \sim 1$ TeV is a mass parameter characterizing the modulus mediation, γ_i are defined in (1.3.39), and $\alpha = \frac{m_{3/2}}{M_0 \ln(M_{Pl}/m_{3/2})} = \mathcal{O}(1)$ is a parameter representing the ratio of anomaly mediation to modulus mediation. Thus the generic mirage mediation is parameterized by

$$M_0,\ \alpha,\ a_i,\ c_i = 1 - n_i,\ \tan\beta, \tag{2.2.20}$$

The parameter values $c_i = a_i = 1$ and $\alpha = 1$ corrensporrespond to the minimal KKLT compactification of type IIB theory with modular weight $n_i = 0$, but other parameter values are also possible for different scenarios, for example choice of $\alpha = 2$ with $a_{H_U} = c_{H_U} = 0$ and $a_{U_3} + a_{Q_3} = c_{U_3} + c_{Q_3} = 0$ can possibly minimize the fine tuning for the electroweak symmetry breaking [86, 87, 88, 89]. At low energies, the gaugino masses in mirage mediation can be written as

$$\frac{M_a(\mu)}{g_a^2(\mu)} = \left(1 + \frac{\ln(\overline{M}_{Pl}/m_{3/2})}{16\pi^2} g_{GUT}^2 b_a \alpha\right) \frac{M_0}{g_{GUT}^2}. \tag{2.2.21}$$

An interesting consequence is that the gaugino masses are unified at a mirage messenger scale [90]

$$M_{\text{mir}} = M_{GUT} \left(\frac{m_{3/2}}{M_{Pl}}\right)^{\alpha/2}, \tag{2.2.22}$$

2.2.5 Deflected mirage mediation

The deflected mirage mediation (DMM) mechanism for supersymmetry breaking mechanism is a mode that is further generalised to include all three known flavor and CP-conserving mediations and involves contributions of comparable scale from gravity mediation, anomaly mediation and gauge mediation [91, 92, 93]. The phenomenological aspects have been investigated in [94, 95, 96, 97]. In DMM the quantity

$$\alpha_m = m_{3/2}/(M_0 \log M_P/m_{3/2}), \tag{2.2.23}$$

parametrises the anomaly to gravity mediation ratio, while M_0 describes the mass scale of soft supersymmetry breaking terms [98]. Here $m_{3/2}$ is the gravitino mass and M_P the reduced Planck mass. The ratio of the gauge mediated contribution to its anomaly mediated counterpart is parametrised by α_g. It is related to the messenger fields by the equation

$$|\alpha_g| = \Lambda/m_{3/2}. \tag{2.2.24}$$

where Λ is a mass scale associated with the messenger fields. The absolute value allows α_g to have negative values. The parameters α_m and α_g can be considered continuous but in string motivated scenarios they usually have discrete values of the order one [99]. The messenger sector is assumed to come in complete GUT representations in order to preserve gauge coupling unification.

N represents the number of copies of $\mathbf{5}, \bar{\mathbf{5}}$ representations under $SU(5)$. The original Kaluza-Klein compactification is obtained with α_m=1 and N =0. Phenomenological implications of various values of the parameters are discussed in e.g. [99] and [100], especially regarding to the Higgs mass.

Above the messenger scale the renormalization group equations are modified from the MSSM form by adding the number of messenger pairs to the β-function coefficients b_a [98]. Thus,

$$b'_a = b_a + N, \tag{2.2.25}$$

where $\{b_1, b_2, b_3\} = \{33/5, 1, -3\}$. At the GUT scale $\mu_{\rm GUT}$, the gaugino mass boundary conditions can be written as [90]:

$$\begin{aligned} M_a\left(\mu_{\rm GUT}\right) =& M_0\left(1 + g_a \ln(M_P/m_{3/2}) b_a g_a \alpha_m\right) \\ =& M_0 + g_a^2\left(\mu_{\rm GUT}\right)\frac{b'_a}{16\pi^2} m_{3/2}, \qquad (a = 1, 2, 3). \end{aligned} \tag{2.2.26}$$

Here μ_{GUT} is the high scale which we take to be the GUT scale. Similarly the scalar masses can be written as

$$\begin{aligned} m_i^2(\mu_{GUT}) =& M_0^2\left[(1 - n_i) - \frac{\theta_i}{16\pi^2}\alpha_m \ln(M_P/m_{3/2}) - \frac{\dot{\gamma}'_i}{(16\pi^2)^2}(\alpha_m \ln(M_P/m_{3/2}))^2\right] \\ =& M_0^2(1 - n_i) - \frac{\theta_i}{16\pi^2} m_{3/2} M_0 - \frac{\dot{\gamma}'_i}{(16\pi^2)^2} m_{3/2}^2, \end{aligned} \tag{2.2.27}$$

where n_i are the modular weights for the scalar masses. The quantities $\dot{\gamma}$, and θ_i are defined in (2.2.19). $\dot{\gamma}'_i$ is obtained by replacing b_a with $b'_a = b_a + N$. For explicit values of θ'_i, $\dot{\gamma}'_i$ see [98]. One-loop renormalization group equations give the boundary condition at the messenger scale $\mu_{\rm mess}$ for the soft gaugino mass parameters:

$$M_a = g_a^2 \frac{b'_a}{16\pi^2} m_{3/2} + M_0\left[1 - g_a^2 \frac{b'_a}{8\pi^2} \log\left(\frac{\mu_{\rm GUT}}{\mu_{\rm mess}}\right)\right] + \Delta M_a \qquad (a = 1, 2, 3), \tag{2.2.28}$$

where

$$\Delta M_a = -N M_0 \frac{g_a^2}{16\pi^2} \alpha_m (1 + \alpha_g) \ln \frac{M_P}{m_{3/2}} \tag{2.2.29}$$

is a threshold contribution that arises when the messenger fields are integrated out. Similarly, scalar masses receive a threshold correction,

$$\Delta m_i^{2j} = M_0^2 \sum_a 2c_a(\Psi_i) N \frac{g_a^4(\mu_{\rm mess})}{(16\pi^2)^2}\left[\alpha_m (1 + \alpha_g) \ln \frac{M_P}{m_{3/2}}\right]^2 \delta_i^j. \tag{2.2.30}$$

The gaugino masses unify at the mirage scale [98]

$$\mu_{\text{mirage}} = \mu_{GUT} \left(\frac{m_{3/2}}{M_P} \right)^{\alpha_{\text{m}}\rho/2}, \tag{2.2.31}$$

in which

$$\rho = \frac{1 + \frac{2Ng_0^2}{16\pi^2} \ln \frac{M_{\text{GUT}}}{\mu_{\text{mess}}}}{1 - \frac{\alpha_{\text{m}}\alpha_{\text{g}}Ng_0^2}{16\pi^2} \ln \frac{M_P}{m_{3/2}}}. \tag{2.2.32}$$

When $\rho = 1$, this reduces to the mirage scale of pure mirage mediation as the deflection is removed. We note that even if gauge mediation is turned off by setting $\alpha_g = 0$, mirage mediation is not recovered. This is achieved only by removing the messenger fields by setting $N = 0$. This is due to the messenger particles affecting the beta functions and thus anomaly mediation at high scales.

Chapter 3

Supersymmetry breaking and the spectrum and decay of neutralinos and charginos

3.1 Introduction

Most of the supersymmetric particles that are likely to be produced at the LHC will not be detected as such, since they will eventually decay into the LSP, which is stable as long as the R-parity is conserved. Thus, the experimental study of supersymmetry involves the study of cascade decays of the supersymmetric particles to the LSP, and the subsequent reconstruction of the decay chains. The LSP in a large class of supersymmetry breaking models is the lightest neutralino, which has, therefore, been a subject of intense study for a long time [101, 102, 103, 104, 105, 106, 107, 10]. A stable lightest neutralino is also an excellent candidate for dark matter [108]. In view of the possible production of supersymmetric particles and their subsequent decay into the lightest neutralino at the LHC, the properties of the lightest neutralino, and also those of heavier neutralinos and charginos, which often appear in the cascade decays, are of considerable importance. In particular a detailed study of the lightest neutralino, especially the predictions for its mass, are of great importance for the supersymmetric phenomenology.

In the MSSM at least two Higgs doublets H_u and H_d with hypercharge (Y) having values -1 and $+1$, respectively, are required to generate masses for all the SM fermions and gauge bosons, and to cancel triangle anomalies. The fermionic partners of these Higgs doublets mix with the fermionic partners of the gauge bosons to produce four neutralino states $\tilde{\chi}_i^0, i = 1, 2, 3, 4$, and two chargino states $\tilde{\chi}_i^{\pm}, i = 1, 2$. In extended supersymmetric models, there can be more Higgs representations, and chargino and neutralino states [103, 104].

The masses of the neutralinos and charginos depend, besides other model parameters, on the soft supersymmetry breaking gaugino masses corresponding to the $SU(2)_L$ and $U(1)_Y$ gauge groups.

The soft gaugino masses provide a handle for identifying the type of supersymmetry breaking [83, 109, 110] in the gaugino sector. The possibility to detect the gaugino mass non-universality at the LHC was studied in [109]. It is because of the distinctive patterns of gaugino masses that one is tempted to believe that neutralinos and charginos are a key in understanding the supersymmetry breaking mechanism.

3.2 Chargino and neutralino masses and gaugino mass patterns

In this section we will describe constraints on the parameters of the neutralino mass matrix which follow from the present experimental limits on the mass of the lightest chargino. We will then discuss the patterns for the gaugino mass parameters that arise in different supersymmetry breaking scenarios, and the resulting consequences for the mass of the lightest neutralino.

3.2.1 Experimental constraints

Collider experiments have searched for the supersymmetric partners of the SM particles. No supersymmetric partners of the SM particles have been found in these experiments. At present only lower limits on their masses have been obtained. The lower limit depends on the spectrum of the model [111]. Assuming that m_0 is large, the limit on the lightest chargino mass following from non-observation of chargino pair production in e^+e^- collisions is

$$M_{\tilde{\chi}_1^{\pm}} \gtrsim 103 \ \ \text{GeV}. \tag{3.2.1}$$

The bound depends on the sneutralino mass. For a sneutralino mass below 200 GeV, the bound becomes weaker, since the production of a chargino pair becomes more rare due to the negative interference between γ or Z in the s-channel and $\tilde{\nu}$ in the t-channel. In the models we consider, $m_{\tilde{\nu}}$ is close to m_0.

When $m_{\tilde{\nu}} < 200$ GeV, but $m_{\tilde{\nu}} > m_{\tilde{\chi}^\pm}$, the limit becomes [111]

$$M_{\tilde{\chi}_1^\pm} \gtrsim 85 \ \ \text{GeV}. \tag{3.2.2}$$

For the parameters of the chargino mass matrix the limit (3.2.1) implies an approximate lower limit [112, 113]

$$M_2, \quad \mu \gtrsim 100 \ \ \text{GeV}. \tag{3.2.3}$$

The limits Eq. (3.2.3) on the parameters M_2 and μ are found by scanning over the MSSM parameter space and are thus model independent.

3.2.2 Gaugino mass patterns

Having constrained the parameters M_2 and μ, which enter the chargino as well as the neutralino mass matrix, we now turn to the theoretical models for the supersymmetry breaking gaugino mass parameters M_1, M_2, and M_3. Theoretically, a simple set of patterns has emerged for these SUSY breaking parameters, which can be described as follows.

Gravity mediated supersymmetry breaking

The first pattern, which has been the object of extensive studies, is the one which arises in the gravity mediated supersymmetry breaking models. In the MSSM with gravity mediated supersymmetry breaking and with a universal gaugino mass $m_{1/2}$ at the grand unified scale (GUT), usually referred to as mSUGRA scenario, we have the boundary conditions ($\alpha_i = g_i^2/4\pi$, $i = 1, 2, 3$)

$$M_1 \ = \ M_2 = M_3 = m_{1/2}, \tag{3.2.4}$$
$$\alpha_1 \ = \ \alpha_2 = \alpha_3 = \alpha_G, \tag{3.2.5}$$

at the GUT scale M_G. The RGE's (2.2.6) and (2.2.7) imply that the soft supersymmetry breaking gaugino masses scale like gauge couplings:

$$\frac{M_1(M_Z)}{\alpha_1(M_Z)} \ = \ \frac{M_2(M_Z)}{\alpha_2(M_Z)} = \frac{M_3(M_Z)}{\alpha_3(M_Z)} = \frac{m_{1/2}}{\alpha_G}, \tag{3.2.6}$$

which implies that M_i/g_i^2 does not run at the one-loop level. Although in the context of the gravity mediated supersymmetry breaking models arbitrary soft gaugino masses are possible, we shall here consider the mSUGRA realisation

(3.2.4) of the gravity mediated supersymmetry breaking scheme. The relation (3.2.6) reduces the three gaugino mass parameters to one, which we take to be the gluino mass $M_{\tilde{g}}$. The other gaugino mass parameters are then determined through

$$M_1(M_Z) = \frac{5\alpha}{3\alpha_3 \cos^2\theta_W} M_{\tilde{g}} \simeq 0.14\ M_{\tilde{g}}, \tag{3.2.7}$$

$$M_2(M_Z) = \frac{\alpha}{\alpha_3 \sin^2\theta_W} M_{\tilde{g}} \simeq 0.28\ M_{\tilde{g}}, \tag{3.2.8}$$

where we have used the value of various couplings at the Z^0 mass

$$\alpha^{-1}(M_Z) = 127.9, \qquad \sin^2\theta_W = 0.23, \qquad \alpha_3(M_Z) = 0.12. \tag{3.2.9}$$

For the gaugino mass parameters this leads to the ratio

$$M_1 : M_2 : M_3 \simeq 1 : 2 : 7.1. \tag{3.2.10}$$

This pattern is typical of any scheme obeying Eqs. (2.2.6) and (3.2.4). Note that the masses above are the running masses evaluated at the electroweak scale, M_Z. This discussion of the gaugino mass parameters is valid at tree level. When radiative corrections are included, the ratio for these parameters in mSUGRA is modified to

$$M_1 : M_2 : M_3 \simeq 1 : 1.9 : 6.2. \tag{3.2.11}$$

Using the ratio (3.2.11) and the lower limit (3.2.3), we have the constraint

$$M_1 \gtrsim 50 \text{ GeV}, \tag{3.2.12}$$

in the gravity mediated supersymmetry breaking models.

Anomaly mediated supersymmetry breaking

Using Eq. (2.2.12), we have the following pattern for the ratio of the gaugino masses at tree level:

$$M_1 : M_2 : M_3 \simeq 3.3 : 1 : 9, \tag{3.2.13}$$

which, after radiative corrections (assuming $m_{3/2} = 40$ TeV) are included, becomes

$$M_1 : M_2 : M_3 \simeq 2.8 : 1 : 7.1, \tag{3.2.14}$$

in the minimal supersymmetric standard model with anomaly mediated supersymmetry breaking. Schemes in which this pattern is realized require a strict separation of hidden sector that breaks SUSY from the visible sector of the MSSM. This implies a strong sequestering, and requires that all supersymmetry breaking fields are sequestered from the visible sector. Nevertheless, it may be achieved in certain class of theories with extra dimensions or a conformal field theory sector.

Using (3.2.3), and the anomaly pattern of the gaugino masses (3.2.14), we have

$$M_1 \gtrsim 280 \text{ GeV}. \tag{3.2.15}$$

This is to be contrasted with the corresponding result (3.2.12) for the gravity mediated supersymmetry breaking.

Mirage mediated supersymmetry breaking

From (2.2.15) and for $g^2_{GUT} \simeq 1/2$ the resulting low energy values yield the mirage mass pattern

$$M_1 : M_2 : M_3 \simeq (1 + 0.66\alpha) : (2 + 0.2\alpha) : (6 - 1.8\alpha). \tag{3.2.16}$$

When the radiative corrections are included for the supersymmetry breaking gaugino masses, we obtain

$$M_1 : M_2 : M_3 \simeq 1 : 1.5 : 2.1 \quad \text{for } \alpha = 1, \tag{3.2.17}$$
$$M_1 : M_2 : M_3 \simeq 1 : 1.2 : 0.92 \quad \text{for } \alpha = 2. \tag{3.2.18}$$

where we have used the value $M_0 = 1$ TeV. Thus, for the mirage mediation, we find

$$M_1 \gtrsim 67 \text{ GeV} \quad \text{for } \alpha = 1, \tag{3.2.19}$$
$$M_1 \gtrsim 83 \text{ GeV} \quad \text{for } \alpha = 2. \tag{3.2.20}$$

Comparison of the patterns of neutralino and chargino masses

In Table 3.1 we show the lightest neutralino and chargino masses, which satisfy the experimental limit for the mass of the lightest chargino [111] for a particular parameter point. These masses are calculated using SOFTSUSY (v.3.0.13) [114]. The absolute value of the Higgsino mixing parameter is determined by the condition of radiative electroweak symmetry breaking (REWSB),

Parameters	mSUGRA	AMSB	Mirage $\alpha = 1$	Mirage $\alpha = 2$
$\tan\beta = 5$ $m_0 = 200$ GeV	(58,105,250,277) (103,278)	(85,245,505,518) (85,518)		
$\tan\beta = 20$ $m_0 = 200$ GeV	(58,104,229,253) (103,253)	(85,237,474,482) (85,484)		
$\tan\beta = 5$ $m_0 = 1$ TeV	(55,103,346,363) (103, 365)	(102,286,629,638) (103,640)	(72,85,165,176) (85,184)	(163,186,473,489) (174,479)
$\tan\beta = 20$ $m_0 = 1$ TeV	(58,104,211,240) (103,242)	(103,286,534,541) (103,545)	(72,94,173,197) (85,202)	(140,161,549,566) (150,553)

Table 3.1: The lower limits on the masses of the four neutralino states and two chargino states in each model in the form $(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0})$ [GeV] followed by $(m_{\chi_1^\pm}, m_{\chi_2^\pm})$ [GeV], with the given set of parameters, following from the experimental lower bound on the mass of the lightest chargino. For the mirage mediation model with $\alpha = 2$ the limit is not from the chargino mass bound, but from the requirement of the nontachyonic spectrum.

and thus depends on the soft scalar mass parameter m_0. The parameter m_0 also enters the radiative corrections through the scalar masses. The masses of the neutralinos in Table 3.1 have been calculated for $m_0 = 200$ GeV and $m_0 = 1$ TeV for both the mSUGRA and AMSB models, and $c_i = a_i = 1$ for the mirage mediation. The sign of the μ parameter was chosen positive. Changing it to the negative value can reduce the neutralino masses by a few GeV's, but this may lead to conflict with the $b \to s\gamma$ constraint. In mSUGRA the trilinear A-parameter was set to zero. Changing that to nonzero values also may decrease the lightest neutralino masses by a few GeV's. These masses demonstrate the effect of the sfermion spectrum on the neutralino and chargino masses for the models that we have studied in [1]. For anomaly mediated supersymmetry breaking, we see the familiar result that the lightest neutralino is closely degenerate with the lightest chargino. The neutralino spectrum in AMSB models is typically heavier than in the case of mSUGRA. In the case of mirage mediation, the mass difference of the lightest and heaviest neutralino masses is smaller as compared to this mass difference in the other models. In addition, especially in the $\alpha = 2$ model, the μ-parameter is smaller as compared to its value in the other models, leading to larger mixing of the gaugino and Higgsino components. The much heavier spectrum in the $\alpha = 2$ mirage mediation model is due to the tachyonic stops in the spectrum for the lighter particles.

The masses of the neutralinos are plotted in Fig. 3.1 for the mSUGRA, AMSB and the mirage mediation scenarios, respectively. In mSUGRA the lightest neutralino is more than 80% bino, while the second lightest one is

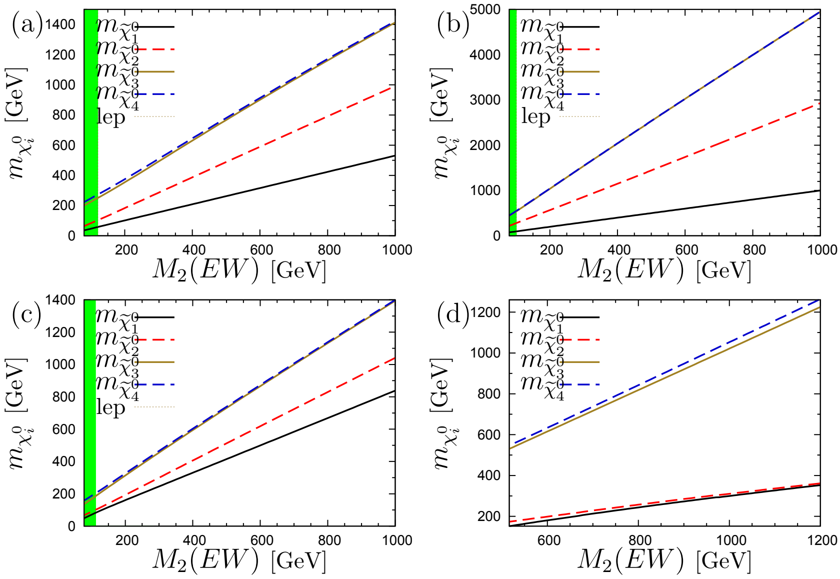


Figure 3.1: Masses of the neutralinos in the (a) mSUGRA, (b) AMSB, and for mirage mediation models with (c) $\alpha = 1$ and (d) $\alpha = 2$. Here $\tan\beta = 10$, $\text{sgn}(\mu) = +1$, $A_0 = 0$, $m_0 = 1$ TeV (mSUGRA) and 1.5 TeV (AMSB). lep-denoted shading indicates the violation of the LEP sparticle mass limits[2].

more than 70% wino. In AMSB, χ_1^0 is almost 100% wino and χ_2^0 bino. In the mirage mediation pattern with $\alpha = 1$, the compositions of the two lightest neutralinos are more evenly divided between bino and wino, but the lightest one is dominantly bino and the second lightest one wino. For small M_2, also the Higgsino component is non-negligible in both. For $\alpha = 2$, the μ-parameter becomes relatively small, and both χ_1^0 and χ_2^0 are more than 90% Higgsinos. However, for the small values of M_2, the LSP is not a neutralino as we will discuss later in Sec. VI (see Fig. 3.9b).

The ratio of the mass parameters, $|M_3|/|M_2|$ is known to differ drastically in different models. When the radiative corrections are taken into account, the

[2] The current LHC limits indicate roughly $M_2 \gtrsim 570$ GeV, 260 GeV, 1330 GeV, 2000 GeV for AMSB, mSUGRA, mirage mediation $\alpha = 1, 2$, respectively (Based on $M_{\tilde{g}} \backsimeq |M_3| \gtrsim 1900$ GeV. See [115]).

ratios for each pattern discussed above is calculated to be

$$\left.\frac{|M_3|}{|M_2|}\right|_{\mathrm{mSUGRA}} = 3.3, \ \left.\frac{|M_3|}{|M_2|}\right|_{\mathrm{AMSB}} = 7.1,$$
$$\left.\frac{|M_3|}{|M_2|}\right|_{\alpha=1} = 1.4, \ \left.\frac{|M_3|}{|M_2|}\right|_{\alpha=2} = 0.77.$$

If one applies the ratio to the masses of the particles, one finds

$$\left.\frac{|m_{\tilde{g}}|}{|m_{\chi_1^{\pm}}|}\right|_{\mathrm{mSUGRA}} = 3.8 - 4.3, \ \left.\frac{|m_{\tilde{g}}|}{|m_{\chi_1^{\pm}}|}\right|_{\mathrm{AMSB}} = 7.3 - 7.4,$$
$$\left.\frac{|m_{\tilde{g}}|}{|m_{\chi_1^{\pm}}|}\right|_{\alpha=1} = 1.7, \qquad \left.\frac{|m_{\tilde{g}}|}{|m_{\chi_1^{\pm}}|}\right|_{\alpha=2} = 0.9,$$

with other parameter values as given in Table 3.1. Thus, very large mass ratio of gluino and chargino hints to an AMSB type breaking, while small value hints towards mirage type of supersymmetry breaking.

3.3 The general upper bound on the mass of the lightest neutralino

In this Section we shall consider a general upper bound on the mass of the lightest neutralino that follows from the structure of the neutralino mass matrix. Since some of the neutralino masses resulting from diagonalization of the mass matrix (1.3.11) can be negative, we shall for our purposes consider the squared mass matrix $\hat{\mathcal{M}}^\dagger\hat{\mathcal{M}}$. This squared mass matrix can be written as

$$\mathcal{M}_0^\dagger\mathcal{M}_0 = \begin{pmatrix} M_1^2 + M_Z^2 s_w^2 & -M_Z^2 c_w s_w & -M_Z s_w(M_1 c_\beta + \mu s_\beta) & M_Z s_w(M_1 s_\beta + \mu c_\beta) \\ -M_Z^2 c_w s_w & M_2^2 + M_Z^2 c_w^2 & M_Z c_w(M_2 c_\beta + \mu s_\beta) & -M_Z c_w(M_2 s_\beta + \mu c_\beta) \\ -M_Z s_w(M_1 c_\beta + \mu s_\beta) & M_Z c_w(M_2 c_\beta + \mu s_\beta) & M_Z^2 c_\beta^2 + \mu^2 & M_Z^2 c_\beta s_\beta \\ M_Z s_w(M_1 s_\beta + \mu c_\beta) & -M_Z c_w(M_2 s_\beta + \mu c_\beta) & -M_Z^2 c_\beta s_\beta & M_Z^2 s_\beta^2 + \mu^2 \end{pmatrix}, \quad (3.3.1)$$

where $c_W = \cos\theta_W, s_W = \sin\theta_W, c_\beta = \cos\beta$ and $s_\beta = \sin\beta$, respectively. An upper bound on the squared mass of the lightest neutralino χ_1^0 can be obtained by using the fact that the smallest eigenvalue of $\mathcal{M}_0^\dagger\mathcal{M}_0$ is smaller than the smallest eigenvalue of its upper left 2×2 sub-matrix

$$\begin{pmatrix} M_1^2 + M_Z^2 \sin^2\theta_W & -M_Z^2 \sin\theta_W \cos\theta_W \\ -M_Z^2 \sin\theta_W \cos\theta_W & M_2^2 + M_Z^2 \cos^2\theta_W \end{pmatrix}, \qquad (3.3.2)$$

thereby resulting in the tree-level upper bound [105]

$$M^2_{\tilde{\chi}^0_1} \leq \frac{1}{2}\left(M_1^2 + M_2^2 + M_Z^2 - \sqrt{(M_1^2 - M_2^2)^2 + M_Z^4 - 2(M_1^2 - M_2^2)M_Z^2 \cos 2\theta_W}\right). \tag{3.3.3}$$

We emphasize that the upper bound (3.3.3) is independent of the supersymmetry conserving parameter μ and also independent of $\tan\beta$, but depends on the supersymmetry breaking gaugino mass parameters M_1 and M_2. Despite this dependence on the unknown supersymmetry breaking parameters Eq. (3.3.3) leads to a useful bound on $M_{\chi_1^0}$. An alternative bound on the mass of the lightest neutralino can be obtained by considering the bottom-right 2×2 submatrix

$$\begin{pmatrix} M_Z^2 \cos^2\beta + \mu^2 & -M_Z^2 \cos\beta \sin\beta \\ -M_Z^2 \cos\beta \sin\beta & M_Z^2 \sin^2\beta + \mu^2 \end{pmatrix}, \tag{3.3.4}$$

leading to an upper bound

$$M^2_{\chi_1^0} \leq |\mu|^2. \tag{3.3.5}$$

This bound, when supplemented by the electroweak symmetry breaking condition

$$\frac{1}{2}M_Z^2 \;\; = \;\; \frac{(m_1^2 - m_2^2 \tan^2\beta)}{(\tan^2\beta - 1)} - |\mu|^2, \tag{3.3.6}$$

leads to the upper bound

$$M_{\chi_1^0} \;\; \leq \;\; |\mu| = \left[\frac{(m_1^2 - m_2^2 \tan^2\beta)}{(\tan^2\beta - 1)} - \frac{1}{2}M_Z^2\right]^{\frac{1}{2}} \tag{3.3.7}$$

The importance of this bound stems from the fact that it relates the two sectors, namely the supersymmetry breaking gaugino masses and the Higgs masses. One can then use the RG evolution for the parameters on the RHS of the bounds to evaluate the bound. Note that the RG equations for the parameters on the RHS of the bounds involve the gaugino masses, and will, therefore, depend on the boundary conditions for the gaugino masses, and hence on the different supersymmetry breaking models for the gaugino mass parameters. In Fig. 3.2 we have plotted the upper limits for the lightest neutralino mass following from (3.3.3) for the different supersymmetry breaking models as a function of $m_{\tilde{\chi}_1^{\pm}}$. For all the four models studied we plot the tree-level masses, and the two-loop radiatively corrected masses calculated using SOFTSUSY

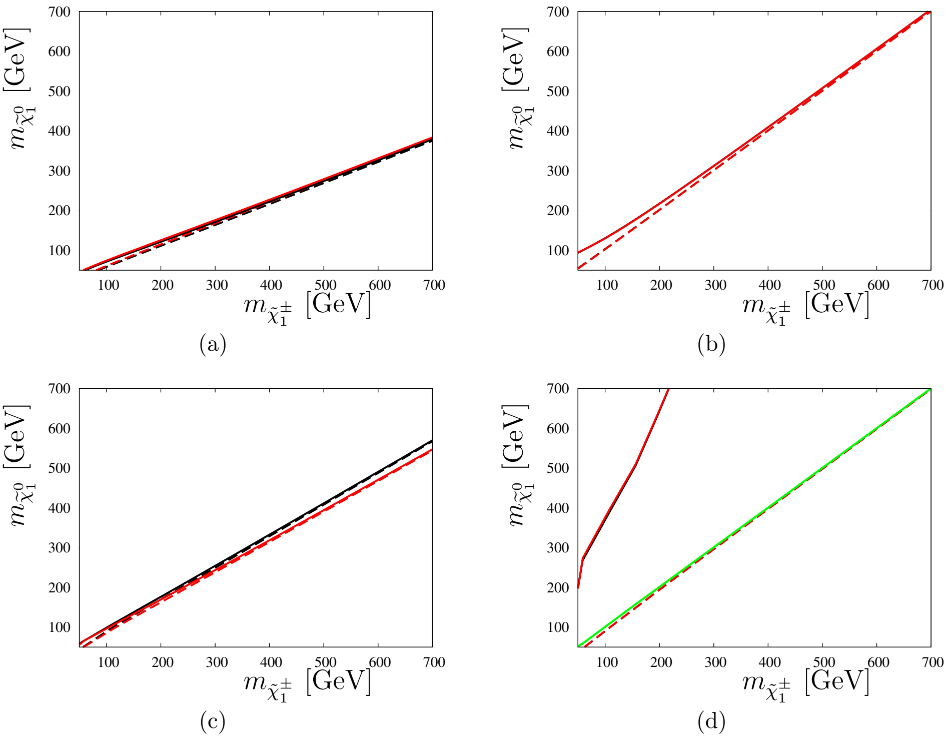


Figure 3.2: Upper limit (solid line) and the mass (dashed line) for the lightest neutralino mass for (a) mSUGRA with $m_0 = 1$ TeV and $A = 0$, (b) AMSB with $m_0 = 1$ TeV, and mirage mediation with $a = c = 1$ and with (c) $\alpha = 1$, and (d) with $\alpha = 2$, calculated at tree-level (red/gray) and with radiative corrections added (black) as a function of the mass of the lighter chargino. In all the plots $\tan\beta = 10$ and $\text{sign}(\mu) = +1$.

(v.3.0.13) [114]. In Fig. 3.2d, we plot also the upper limit from Eq. (3.3.7), which gives the lowest mass upper bound in this case. Note that the mass of the lighter of the charginos is close to the value of the μ-parameter. The neutralino masses have been calculated assuming $\tan\beta = 10$, and the other parameters as indicated in the Figure. We note that all gaugino masses can receive radiative corrections up to 20%, and, thus, difference between tree level and radiatively corrected neutralino and chargino masses can be significant in all models studied in [1], although the difference is not explicit in Fig. 3.2 due to similar magnitude of correction for both the neutralino and the chargino.

3.4 Sum rules

We recall that in the minimal AMSB model, the mass difference between the lightest chargino and the lightest neutralino is small. The close proximity of the lightest neutralino and chargino masses is a direct consequence of Eq. (2.2.12), which gives for the ratios of the gaugino mass parameters $|M_1| : |M_2| : |M_3| \simeq 2.8 : 1 : 7.1$, after taking into account the next to leading order radiative corrections and the weak scale threshold corrections [67] as in (3.2.14). Thus, the winos are the lightest neutralinos and charginos, and one would expect that the lightest chargino is only slightly heavier than the lightest neutralino in all anomaly mediated models. It is not feasible to obtain sum rules for the masses of the neutralino states, since the physical neutralino mass matrix is a 4×4 matrix. However, from the trace of the squares of the neutralino and chargino mass matrices, one obtains a sum rule, which does not contain the Higgs mixing parameter μ, but which is present in the mass matrices. The sum rule can be written as

$$2\sum_{i=1}^{2} M^2_{\tilde{\chi}_i^{\pm}} - \sum_{i=1}^{4} M^2_{\tilde{\chi}_i^{0}} \;=\; \left[M_2^2 - M_1^2\right] + 4M_W^2 - 2M_Z^2. \qquad (3.4.1)$$

By using the gaugino mass pattern for a specific model the sum rule (3.4.1) can be expressed as function of any of the gaugino masses. This is shown in Fig. 3.3. The average mass difference in the AMSB is first positive, but then quickly turns negative, while in the minimal SUGRA model it is always positive. In the mirage mediation model the behavior is determined by the parameter α with a low value leading to a mSUGRA-like curve. Increasing α decreases the gradient of the curve until $\alpha = 2.17$ (the value for which $M_1 = M_2$) leads to a constant positive value. We note that this sum rule could

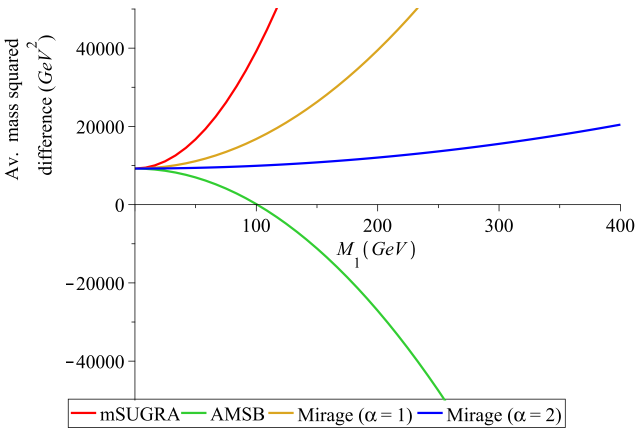


Figure 3.3: The sum rule (3.4.1) plotted for different gaugino mass patterns.

be used as a signature for different supersymmetry breaking models, and in the case of mirage mediation it might be useful for determining the value of α, which can be calculated from the sum rule for specific values of gaugino masses.

3.5 Decays of neutralinos and charginos

In this Section we discuss the decays of charginos and neutralinos in different supersymmetry breaking models that we have discussed in [1]. As noted earlier, charginos and neutralinos are mass eigenstates, which are model-dependent linear combinations of charged or neutral gauginos and Higgsinos. Since the mass matrices of charginos and neutralinos depend on parameters M_1 and M_2, which are model dependent, the decays will depend on the model under consideration. As such the decay patterns of charginos and neutralinos can serve as a window on the underlying supersymmetry breaking mechanism in the gaugino sector. Here we shall mostly focus on two-body tree-level decays of neutralinos and charginos, if they are kinematically possible. If the neutralino or the chargino is sufficiently heavy, then two-body decays into a W, Z^0, or a Higgs boson and a lighter neutralino or chargino are the dominant decay modes.

Since the lightest Higgs boson is relatively light, the two-body decay containing the light Higgs boson is expected to be the dominant decay mode over a large region of parameter space.

However, if some squarks or sleptons are relatively light, the two body tree-level decays of a heavy neutralino or chargino to quark-squark or lepton-slepton may be important. However, these decays are phenomenologically less important at a hadron collider like LHC, where neutralinos and charginos would be produced from the decays of strongly interacting squarks and gluinos. Neutralinos and charginos, which are heavier than squarks, would be hard to study at a hadron collider.

We recall that if a heavier $\tilde{\chi}_i^0$ $(i = 2, 3, 4)$ or a chargino $\tilde{\chi}_j^+$ $(j = 1, 2)$ is produced at a collider, it will decay via a cascade until the lightest neutralino ($\tilde{\chi}_1^0$) is produced. Thus, we are interested in the branching ratios for the two-body decays

$$\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 + Z^0, \quad \tilde{\chi}_i^0 \to \tilde{\chi}_j^\pm + W^\mp, \quad \tilde{\chi}_i^+ \to \tilde{\chi}_j^0 + W^+, \quad \tilde{\chi}_i^+ \to \tilde{\chi}_j^+ + Z^0, \tag{3.5.1}$$
$$\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 + H_k^0, \quad \tilde{\chi}_i^0 \to \tilde{\chi}_j^\pm + H^\mp, \quad \tilde{\chi}_i^+ \to \tilde{\chi}_j^0 + H^+, \quad \tilde{\chi}_i^+ \to \tilde{\chi}_j^+ + H_k^0. \tag{3.5.2}$$

These two body decays will dominate any tree-level three-body decays mediated by virtual squark or slepton exchange. These decays will also dominate any two-body decay, which is forbidden at the tree level, but which can proceed via loops, such as $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 + \gamma$.

If some of the neutralinos and charginos are heavier than some of the squarks and sleptons, then the two-body decays

$$\tilde{\chi} \to q\,\tilde{q},\ l\,\tilde{l} \tag{3.5.3}$$

can compete with the two-body decays into W, Z^0, H discussed above. The analytical expressions for the branching ratios of charginos and neutralinos into W, Z^0, and Higgs bosons, as well as into squarks/sleptons for arbitrary neutralino and chargino mixing angles are given in [116].

For the evaluation of branching ratios of charginos and neutralinos, we have calculated the spectra of the supersymmetric particles using SOFTSUSY (v.3.0.13) [114], and the decays of the supersymmetric particles using SUSY-HIT(v.1.3 with SDECAY v1.3b/HDECAY v3.4) [117]. While calculating the decay rates of charginos and neutralinos, we have imposed the experimental constraints following from LEP sparticle mass limits and the LEP lower bound on the lightest Higgs boson mass. After original publication of the results in [1], the Higgs mass has been determined to be 125 GeV. This fixes the squark mass on the x-axis in our figures to a specific value depending on the model in question. In addition, the new limits for the sparticle masses affect the physical region in the figures. These changes do not affect the main conclusions of our analysis.

In the analysis of the branching ratios we have shown that the second lightest neutralino and the lighter chargino are produced in large amounts in squark decays. This is interesting, since a promising signal to detect weakly interacting particles at Tevatron and at LHC is considered to be the associated production $\tilde{\chi}_1^{\pm}\tilde{\chi}_2^0$, see *e.g.* [118, 119] and references therein. Let us consider the productions of $\tilde{\chi}_1^{\pm}$, $\tilde{\chi}_2^0$ in view of the cascade decays in Figs. (3.4)-(3.7). It is seen that in the studied breaking patterns the largeness of the trilepton signal varies significantly. In the mSUGRA pattern, $\tilde{t}_1$ decays to all the heavier neutralinos and charginos with nonnegligible branching fractions. The contribution $\tilde{t}_1 \to \tilde{\chi}_1^+ b/\tilde{\chi}_2^0 t$ is at a few percent level, but more events come from the decays of $\tilde{\chi}_{3,4}^0$, $\tilde{\chi}_2^+$. Thus from $\tilde{t}_1\bar{\tilde{t}}_1$ production there is an additional contribution to the trilepton signal, accompanied by a number of jets. In the AMSB pattern, the enhancement of trileptons is significant. $\tilde{t}_1$'s decay 60% of the time to $\chi_2^0 t$ and 20 % of the time to $\chi_1^+ b$. As soon as kinematically possible, the χ_2^0 decays to a slepton and lepton, and χ_1^+ decays leptonically 25% of the time. In mirage pattern, stops tend to decay directly to the lightest neutralino and no enhancement is expected.

3.6 Relic density

We have studied the implications of different patterns of gaugino masses for the relic density of lightest neutralino, and the constraints imposed on the parameter space by the precise limits on the relic density obtained by the Wilkinson Microwave Anisotropy Probe (WMAP) satellite. Requiring the lightest neutralino to form all of the dark matter as a thermal relic is very limiting constraint on the parameter space, and it should be kept in mind that the possible dark matter might also be created non-thermally or the excess thermal production diluted for example by an entropy increase after the freeze-out [120, 121, 122]. Therefore we refer to the WMAP constrained parameter space as a WMAP-preferred relic density area.

The relic density in the mSUGRA [123, 124, 125, 126, 127, 128, 129] and AMSB [67, 130, 131, 132, 133] scenarios has been studied extensively. Neutralino dark matter in the mirage mediation SUSY breaking model has been considered in [70, 134, 135, 136]. In the analysis we consider the combined

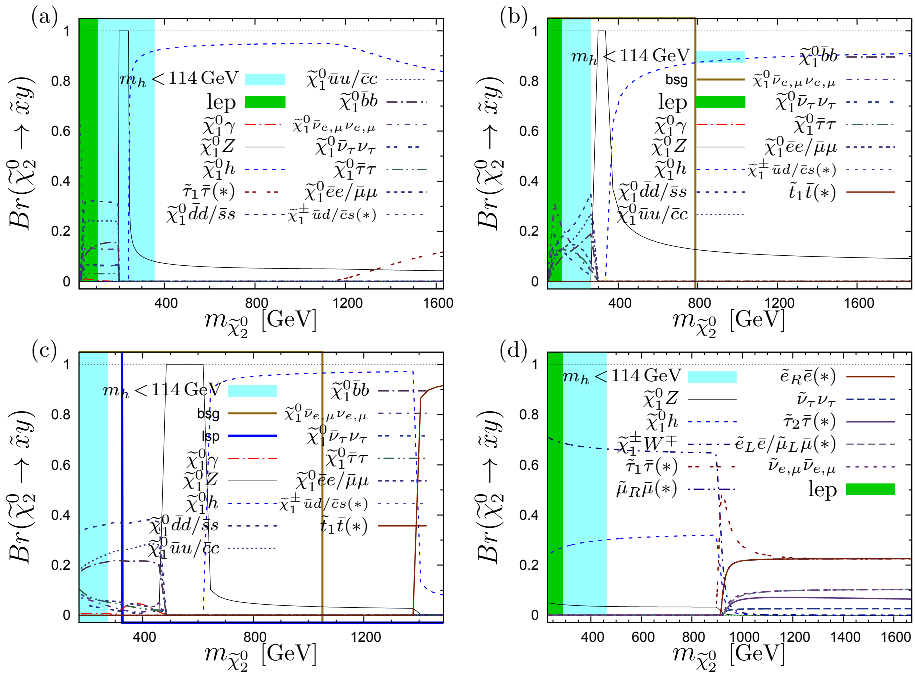


Figure 3.4: Branching ratios for the two body decays of $\tilde{\chi}_2^0$ in (a) mSUGRA model, (b) mirage mediation scenario with $\alpha = 0.5$ and (c) $\alpha = 1$, and (d) the AMSB scenario. Also the three body channels are shown where no two body decays are possible. The $(*)$ in the channel label signifies that the channel includes also the charge conjugated final state. Shadings specify the region where the LEP mass limits are not satisfied (dark) or the lightest Higgs mass is below 114 GeV (light).

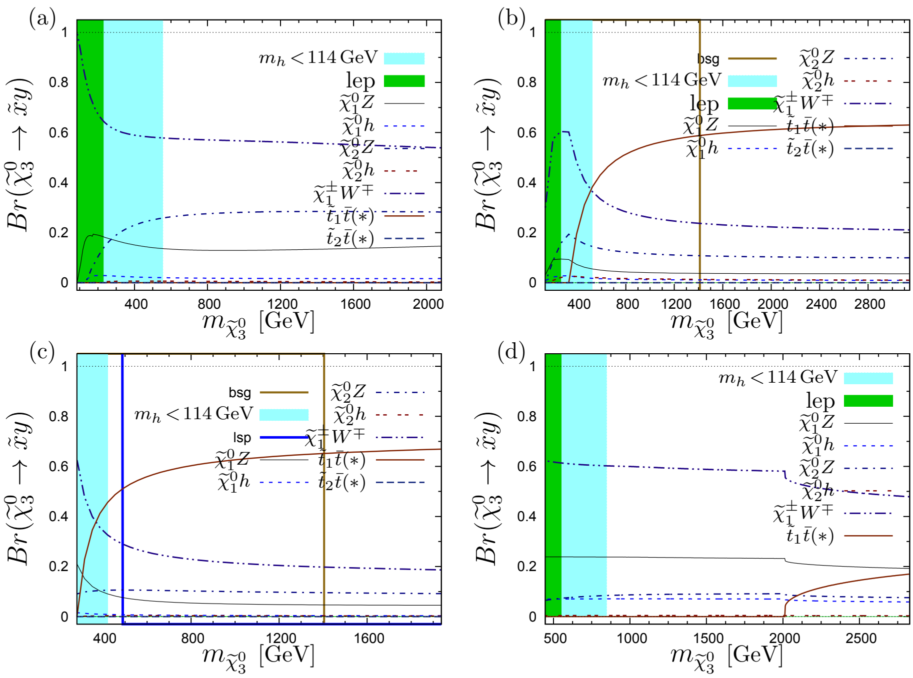


Figure 3.5: Branching ratios for the decays of $\widetilde{\chi}_3^0$ in (a) mSUGRA model, (b) mirage mediation scenario with $\alpha = 0.5$ (c) in the mirage mediation scenario with $\alpha = 1$, and (d) the AMSB scenario.

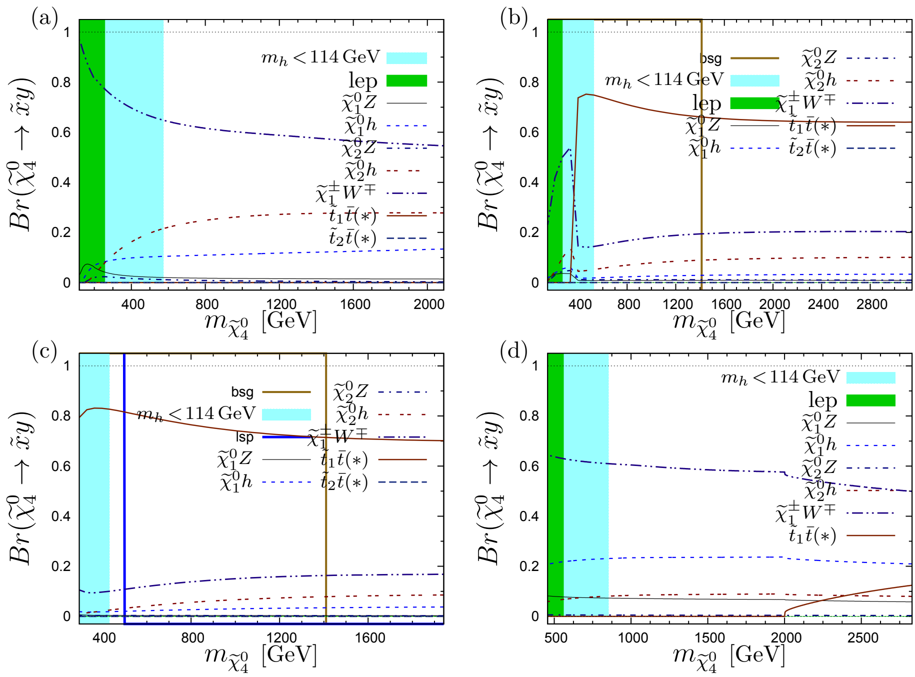


Figure 3.6: Branching ratios for the decays of $\widetilde{\chi}_4^0$ in (a) mSUGRA model, (b) mirage mediation scenario with $\alpha = 0.5$ and (c) $\alpha = 1$, and (d) the AMSB scenario.

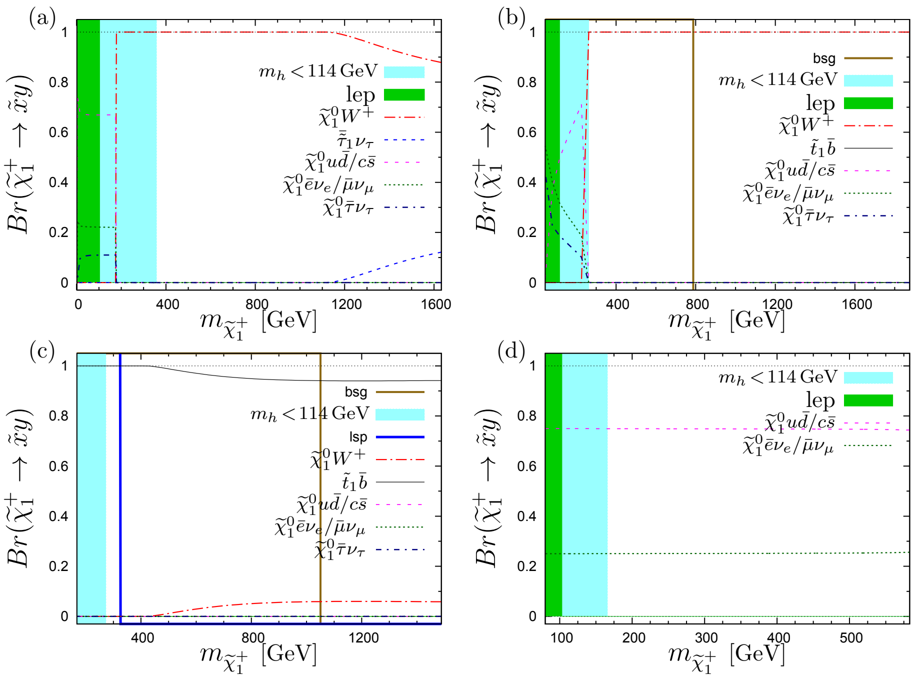


Figure 3.7: Branching ratios for the decays of $\widetilde{\chi}_1^+$ in (a) mSUGRA model, (b) mirage mediation scenario with $\alpha = 0.5$ and (c) $\alpha = 1$, and (d) the AMSB scenario.

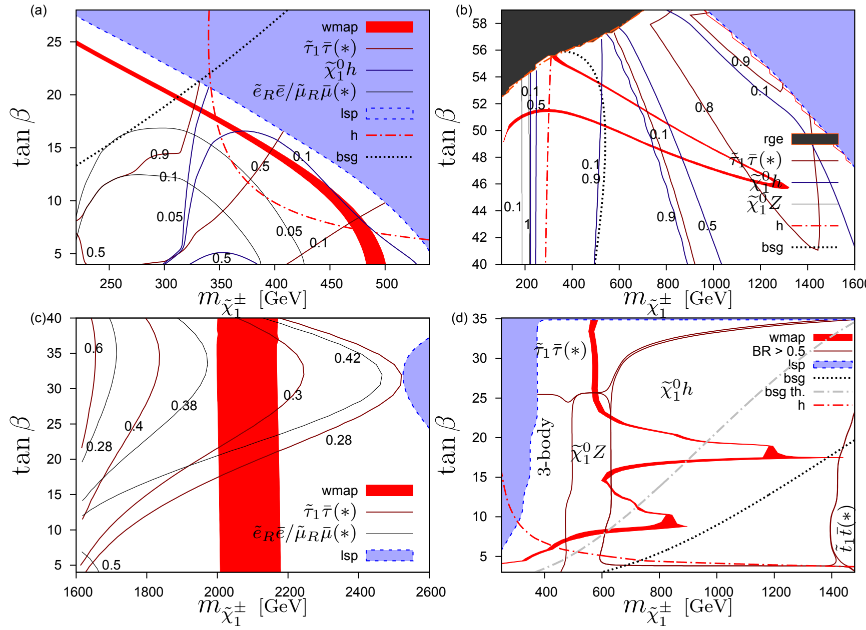


Figure 3.8: Contours of constant branching ratio for the leading two-body decay modes of $\widetilde{\chi}_2^0$ superposed on the same plot with several constraints for (a) mSUGRA scenario for $m_0 = 120$ GeV, (b) $m_0 = 1$ TeV, (c) AMSB for $m_0 = 5$ TeV and (d) the mirage mediation for $\alpha = 1$. The $b \to s\gamma$ constraint is obeyed right of the dotted bsg-denoted line and the lightest Higgs mass is more than 114 GeV on the right of the h denoted dash-dotted line.

information from the relic density and decay modes of the second lightest neutralino. The measurement of Higgs mass fixes the chargino mass as a function $\tan\beta$ but does not affect the main conclusions of our analysis.

We have found that while in the mSUGRA model typically a narrow range with the observed relic density occurs, in the AMSB model the relic density remains below the WMAP limit for the sub-TeV scale spectrum. In mirage mediation models the observed dark matter range is narrow and close to the stop LSP region. We note that it is not necessary that neutralino is the only dark matter particle, even if it were the lightest supersymmetric particle. Furthermore, it is possible that the R-parity is broken at least slightly in nature. This would lead to the neutralino decay, even if the breaking were so tiny that it would not show up in the experiments.

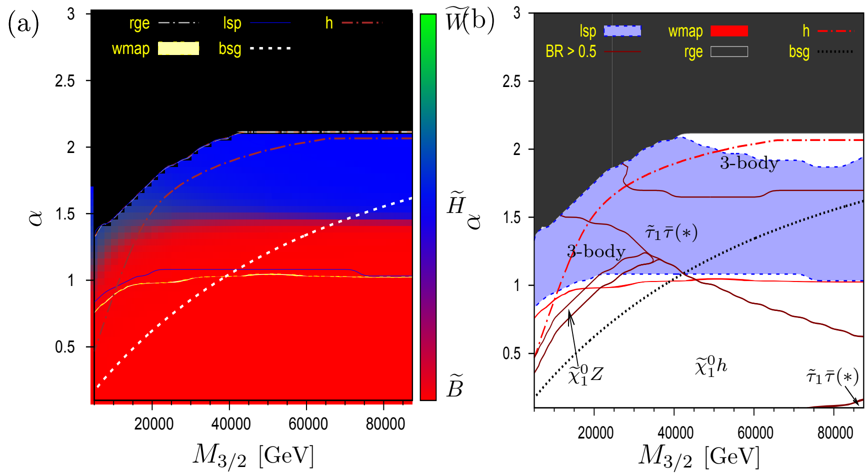


Figure 3.9: Lightest neutralino composition (a) and the leading $\widetilde{\chi}_2^0$ decay modes (b) in the mirage mediation scenario in $(M_{3/2}, \alpha)$ plane for $\text{sgn}(\mu) = +1$, $\tan\beta = 10$ and $a_i = c_i = 1$. The narrow light yellow band in (a) (red in (b)) indicates the WMAP preferred relic density area. The $b \to s\gamma$ constraint is obeyed below the dotted bsg-denoted line and the lightest Higgs mass is more than 114 GeV below the h denoted dash-dotted line. The lsp denoted (light blue) line near the WMAP filling limits the area, above which the lightest neutralino is not the LSP except for the area near $\alpha = 2$, which can better be seen in (b). The black areas limited by the rge-denoted line depicts the area where there are either tachyons or no REWSB. In (b) the domains of branching ratio exceeding 50 % for the leading decay modes of $\widetilde{\chi}_2^0$ are drawn for the same parameters, including the constraints.

3.7 Beyond MSSM operators and neutralino and chargino masses

In [2] we investigated the contribution of the dimension five BMSSM operator (1.3.31) to the neutralino and chargino masses in different SUSY breaking models.

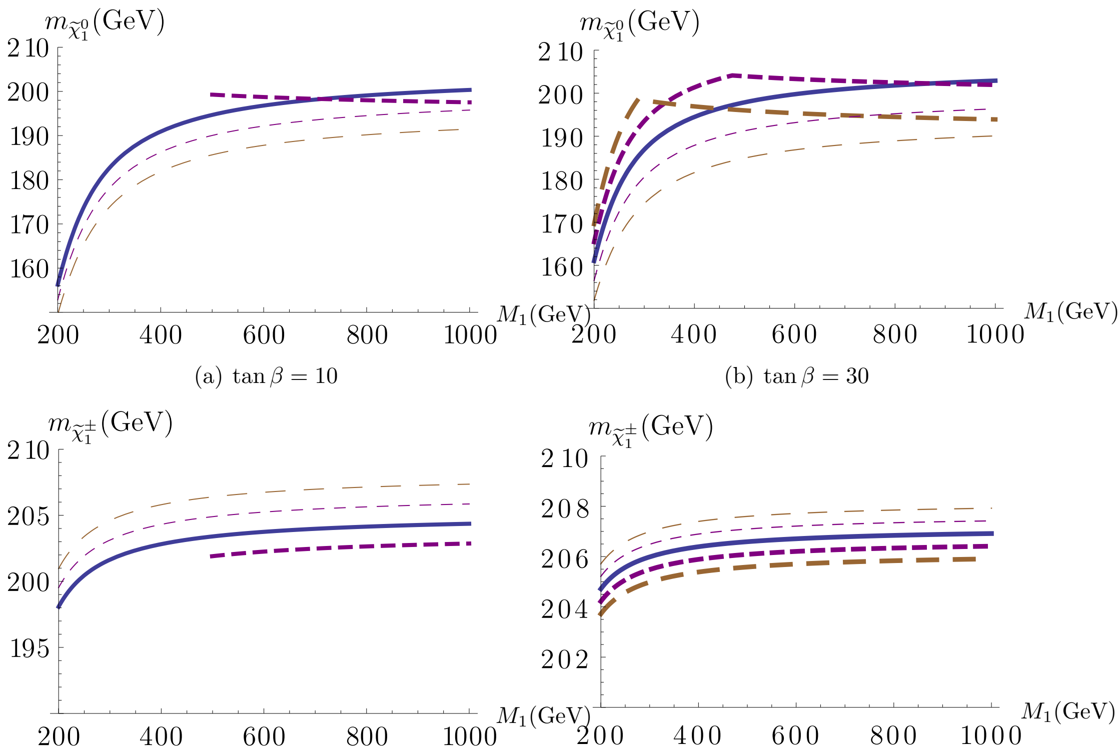
For large values of μ, the lightest neutralino and chargino are almost pure gauginos. In this case, the corrections to the lightest neutralino and chargino masses from BMSSM operators are small, since they affect the Higgsino sector. If, on the other hand, the μ parameter is small compared to the gaugino mass parameters, *i.e.* if the lightest neutralino and chargino are dominantly Higgsinos, the BMSSM corrections to their masses can be significant.

In Fig. 3.10 we show the lightest neutralino and chargino masses for several values of ϵ_1, $\epsilon_1 = 0, \pm 0.05, \pm 0.1$. We have plotted these masses for the mSUGRA model. We have excluded parts of the graphs in Figs. 3.10 - 3.13, where $m_{h^0} < 111$ GeV, which is the Higgs mass limit at the time the calculations were performed. Naturally, since the Higgs mass is now fixed, usefulness of the plots are reduced, but perhaps they are still be instructive in describing the general behaviour of the quantities.

Because for $\mu << M_1, M_2$ the Higgsino sector strongly dominates the lightest neutralino and chargino masses, and thus the plot for mSUGRA is a representative for the mirage mediation models as well since the only difference in the masses in these models is due to the gaugino non-universality. We have not shown the results for the AMSB case, since in the AMSB μ cannot be smaller than M_1 due to the electroweak symmetry breaking condition [67], and thus in this case the dimension five contribution is negligible to the lightest neutralino and chargino masses.

If the μ parameter is large compared to the soft gaugino masses, the two heaviest of the neutralinos are mostly Higgsinos. The relative contribution of the dimension five operator to the mass for a heavy particle is small. If dimension 5 contribution to the masses of neutralinos and charginos is sizable, it is more difficult to determine the supersymmetry breaking mechanism with mass measurements. As an additional tool for distinguishing the effect of SUSY breaking mechanism and the dimension five operator we consider here two different sum rules involving neutralino and chargino masses and their squares. The dependence on gaugino masses enters these sum rules in a specific manner.

From the trace of the neutralino mass matrix (1.3.11) one obtains the sum over the neutralino mass eigenvalues which we denote by σ. This can be written



(a) $\tan\beta = 10$ (b) $\tan\beta = 30$

(c) $\tan\beta = 10$ (d) $\tan\beta = 30$

Figure 3.10: The lightest neutralino and chargino masses in mSUGRA for several values for the parameter $\epsilon_1 = \frac{\lambda}{M}\mu$. The blue solid line corresponds to $\epsilon_1 = 0$, and the thick dashed lines in order of increasing dash length represent $\epsilon_1 = 0.05$ (violet), $\epsilon_1 = 0.1$ (ochre). The thin dashed lines denote the lightest neutralino mass for $\epsilon_1 = -0.05$ (violet), $\epsilon_1 = -0.1$ (ochre), again in the order of increasing dash length. Here $\mu = 200$ GeV and one-loop radiative corrections are included.

as

$$\sigma(\epsilon_1) \equiv \sum_{i=1}^{4} \eta_i m_{\tilde{\chi}_i^0} = M_1 + M_2 + 2\frac{\epsilon_1}{\mu} v^2, \tag{3.7.1}$$

at leading order in ϵ_1, where η_i is the sign of the ith eigenvalue. This sum rule depends on the μ parameter through BMSSM operators, when ϵ_1 is taken as an independent parameter. An advantage of this sum rule is that in addition to the gaugino mass parameters and ϵ_1, it depends only on the supersymmetric Higgsino mixing parameter μ. Using relations (3.2.6), (2.2.12), (2.2.21), and (2.2.22) the gaugino mass parameters M_1 and M_2 can be expressed in terms of the gluino mass $M_{\tilde{g}}$ and coupling constant α_i, both observable quantities. For mSUGRA, AMSB and mirage mediation the sum rule can then be written as, with $B = \ln(M_{GUT}/M_{mir})/(16\pi^2)$,

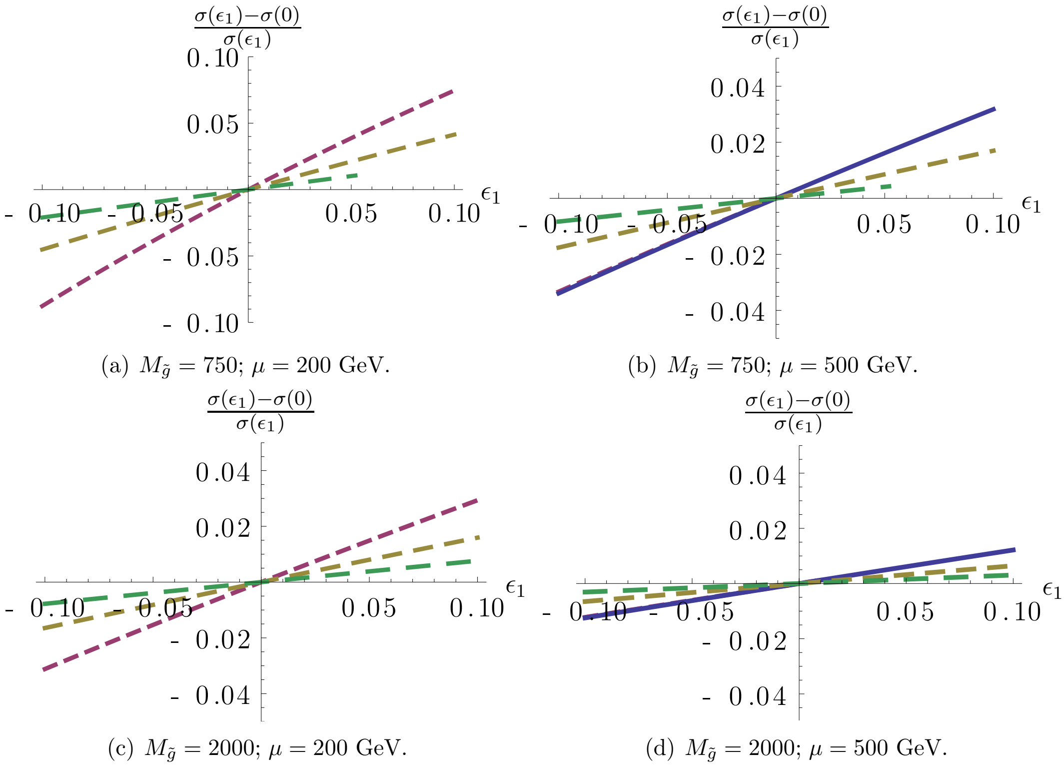
$$\sigma_{mSUGRA}(\epsilon_1) = \frac{M_{\tilde{g}}}{\alpha_3}(\alpha_1 + \alpha_2) + 2\frac{\epsilon_1}{\mu} v^2, \tag{3.7.2}$$

$$\begin{aligned} \sigma_{AMSB}(\epsilon_1) &= \frac{M_{\tilde{g}}}{3}\left[\frac{\alpha_2}{\alpha_3} + \frac{33}{5}\frac{\alpha_1}{\alpha_3}\right] + 2\frac{\epsilon_1}{\mu} v^2, \\ \sigma_{mirage}(\epsilon_1) &= \frac{M_{\tilde{g}}}{\alpha_3}\left[1 - 3B\right]^{-1}\left[\alpha_2\left(1+B\right) + \alpha_1\left(1 + \frac{33}{5}B\right)\right] \\ &\quad + 2\frac{\epsilon_1}{\mu} v^2. \end{aligned} \tag{3.7.3}$$

In Fig. 3.11 we have plotted the magnitude of the dimension five contribution relative to the whole sum with two μ and $M_{\tilde{g}}$ values, $\mu = 200, 500$ GeV, and $M_{\tilde{g}} = 750, 2000$ GeV. The plotted quantities can be written in terms of observables as

$$\frac{\sigma(\epsilon_1) - \sigma(0)}{\sigma(\epsilon_1)} = \frac{\sum_{i=1}^{4} \eta_i m_{\tilde{\chi}_i^0} - \gamma_{SB} M_{\tilde{g}}}{\sum_i \eta_i m_{\tilde{\chi}_i^0}}, \tag{3.7.4}$$

where γ_{SB} refers to the coefficient of $M_{\tilde{g}}$ in different gaugino mass patterns in Eq. (3.7.2). AMSB is not allowed for the $\mu = 200$ GeV case due to the constraint $\mu > M_1$ in this model. In the sum σ the dimension five contribution is inversely proportional to μ, and the maximum percentage contribution is achieved with the lowest gluino mass. The contribution is largest for mSUGRA pattern, and smallest for mirage mediation with $\alpha = 2$. In our example with



(a) $M_{\tilde{g}} = 750$; $\mu = 200$ GeV.

(b) $M_{\tilde{g}} = 750$; $\mu = 500$ GeV.

(c) $M_{\tilde{g}} = 2000$; $\mu = 200$ GeV.

(d) $M_{\tilde{g}} = 2000$; $\mu = 500$ GeV.

Figure 3.11: The contribution arising from ϵ_1 to the total sum of (3.7.1) in different supersymmetry breaking models. The solid blue line corresponds to AMSB; mSUGRA (violet), and mirage mediation with $\alpha = 1$ (ochre), and $\alpha = 2$ (green) models, respectively, are presented in the order of increasing dash length.

$M_{\tilde{g}} = 750$ GeV and $\mu = 200$ GeV, the contribution with $\epsilon_1 = -0.1$ varies between -2.5 % and -9 % .

From the trace of the squares of the neutralino and chargino mass matrices, one obtains a sum rule for the neutralino and chargino masses squared, which we denote by Σ:

$$\begin{aligned}\Sigma(\epsilon_1) &\equiv 2\sum_{i=1}^{2} m^2_{\tilde{\chi}^\pm_i} - \sum_{i=1}^{4} m^2_{\tilde{\chi}^0_i} \\ &= \left[M_2^2 - M_1^2\right] + 4M_W^2 - 2M_Z^2 + 4\epsilon_1 v^2 \sin 2\beta. \end{aligned} \tag{3.7.5}$$

at leading order in ϵ_1. This sum rule depends on $\tan\beta$ in addition to M_1, M_2 and ϵ_1 but not on μ. In this sense the sum rules (3.7.1) and (3.7.5) are complementary.

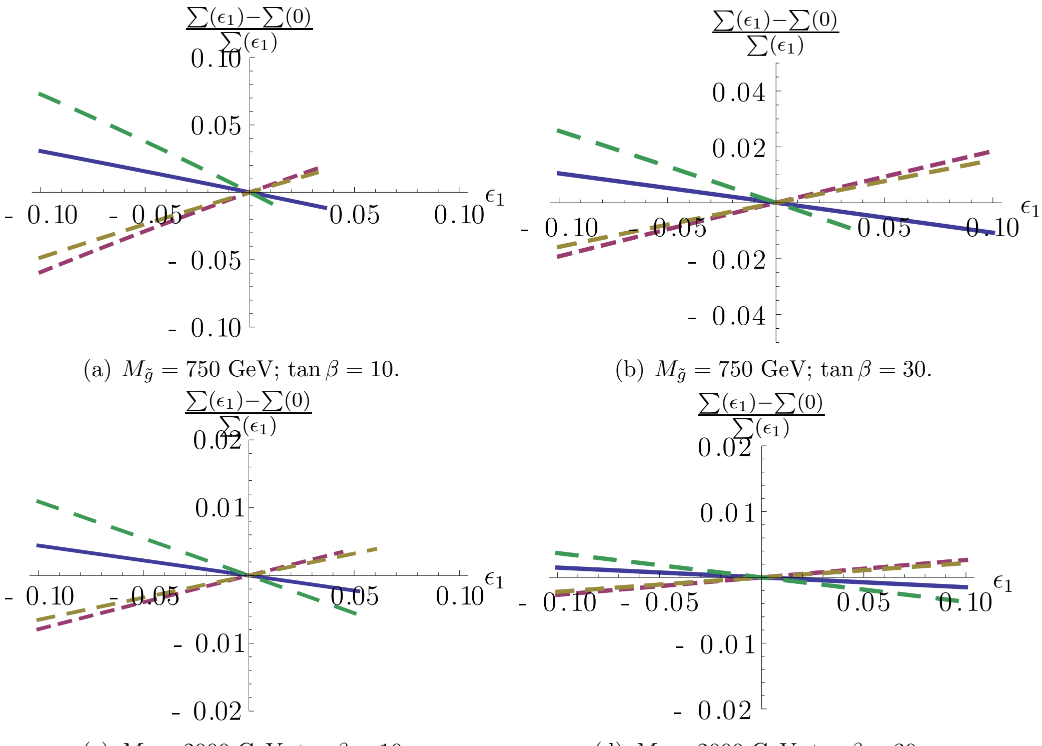
The dimension 5 contribution in $\Sigma(\epsilon_1)$ decreases for increasing $\tan\beta$. The gaugino mass parameters M_1 and M_2 can again be expressed in terms of the gluino mass $M_{\tilde{g}}$ and coupling constants α_i. For mSUGRA, AMSB and mirage mediation the sum rule can be written as

$$\begin{aligned}\Sigma_{mSUGRA}(\epsilon_1) &= \frac{M_{\tilde{g}}^2}{\alpha_3^2}(\alpha_2^2 - \alpha_1^2) + 4M_W^2 - 2M_Z^2 + 4\epsilon_1 v^2 \sin 2\beta, \\ \Sigma_{AMSB}(\epsilon_1) &= \frac{M_{\tilde{g}}^2}{9}\left[\frac{\alpha_2^2}{\alpha_3^2} - (\frac{33}{5})^2\frac{\alpha_1^2}{\alpha_3^2}\right] + 4M_W^2 - 2M_Z^2 + 4\epsilon_1 v^2 \sin 2\beta, \\ \Sigma_{mirage}(\epsilon_1) &= \frac{M_{\tilde{g}}^2}{\alpha_3^2}\left[1-3B\right]^{-2}\left[\alpha_2^2\left(1+B\right)^2 - \alpha_1^2\left(1+\frac{33}{5}B\right)^2\right] \\ &\quad +4M_W^2 - 2M_Z^2 + 4\epsilon_1 v^2 \sin 2\beta. \end{aligned} \tag{3.7.6}$$

In Fig. 3.12 we have plotted the magnitude of the dimension five contribution relative to the whole sum with two $\tan\beta$ and $M_{\tilde{g}}$ values, $\tan\beta = 10, 30$ and $M_{\tilde{g}} = 750, 2000$ GeV. The plotted quantities can be written in terms of observables as

$$\frac{\Sigma(\epsilon_1) - \Sigma(0)}{\Sigma(\epsilon_1)} = 2\frac{\sum_{i=1}^{2} m^2_{\tilde{\chi}^\pm_i} - \sum_{i=1}^{4} m^2_{\tilde{\chi}^0_i} - \alpha_{SB}^2 M_{\tilde{g}}^2}{\sum_{i=1}^{2} m^2_{\tilde{\chi}^\pm_i} - \sum_{i=1}^{4} m^2_{\tilde{\chi}^0_i}}, \tag{3.7.7}$$

where α_{SB} is the supersymmetry breaking model dependent coefficient of $M_{\tilde{g}}^2$ in (3.7.6). As seen from Fig. 3.12 increasing $\tan\beta$ from 10 to 30 rougly halves



(a) $M_{\tilde{g}} = 750$ GeV; $\tan\beta = 10$.

(b) $M_{\tilde{g}} = 750$ GeV; $\tan\beta = 30$.

(c) $M_{\tilde{g}} = 2000$ GeV; $\tan\beta = 10$.

(d) $M_{\tilde{g}} = 2000$ GeV; $\tan\beta = 30$.

Figure 3.12: The contribution arising from ϵ_1 to the total sum of (3.7.5) in different supersymmetry breaking models. The solid blue line corresponds to AMSB; mSUGRA (violet), and mirage mediation with $\alpha = 1$ (ochre), and $\alpha = 2$ (green) models, respectively, are presented in the order of increasing dash length.

the dimension five contribution. Larger $\tan\beta$ however allows larger positive values ϵ_1 without violating the Higgs mass constraint. In contrast with σ, the maximum dimension five contribution of 10 % is seen in the mirage mediation model with $\alpha = 2$, and in mSUGRA the contribution is the lowest of the four examined models. It is seen that for AMSB and mirage mediation with $\alpha = 2$ the contribution to σ is opposite sign to the contribution to Σ, while for mSUGRA and mirage mediation with $\alpha = 1$, σ and Σ have the same sign.

By combining the sum rules Eq. (3.7.2) and (3.7.6) we obtain a relation for $\tan\beta$ and μ that is independent of ϵ_1,

$$\mu = \frac{2\sum_{i=1}^{2} m^2_{\tilde{\chi}^\pm_i} - \sum_{i=1}^{4} m^2_{\tilde{\chi}^0_i} - \alpha^2_{SB} M^2_{\tilde{g}} - 4M_W^2 + 2M_Z^2}{\sum_{i=1}^{4} \eta_i m_{\tilde{\chi}^0_i} - \gamma_{SB} M_{\tilde{g}}} \frac{1+\tan^2\beta}{4\tan\beta}. \quad (3.7.8)$$

This relation can be used for estimating the value of μ in BMSSM models if $\tan\beta$ is known. It should be noted that this formula does not exist without the BMSSM operator ϵ_1. Thus a consistent value with other measurements may indicate the existence of the BMSSM operators. From precise measurements the value of ϵ_1 can also be determined from Eq. (3.7.2) and (3.7.6) when μ or $\tan\beta$ are known.

The gaugino mass pattern realized in Nature may well turn out to be a mixture of the patterns studied here. This possibility can be considered by a general study of the ratio of M_1 and M_2. In Fig. 3.13 we show the fraction of the contribution from the dimension five operator to the sum rule (3.7.5) for $\epsilon_1 = -0.1$ as a function of the ratio of the mass parameters M_2 and M_1. Although at $M_1 = 400$ GeV (and larger) the dimension five contribution remains at less than a few percent for all models, $M_1 = 100$ GeV can produce as high as a 20 percent dimension five contribution in mirage mediation with $\alpha = 2$ and a 10 percent contribution in mSUGRA. As expected, the contribution is highest near the point $M_2/M_1 = 1$, where the sum of the squares of the gaugino mass parameters cancels in the sum rule, thus making the sum completely independent of the gaugino masses. This point corresponds to mirage mediation with $\alpha = 2.17$. Consequently, mirage mediation models with α close to this value allow significant dimension five contributions, although the lower bound for the gluino mass restricts M_1 to 1 TeV range and above. The experimental limit for the chargino mass rules out M_1 lower than 280 GeV in AMSB, and the dimension five contribution remains at a few percent for all allowed values for the gaugino masses for this model.

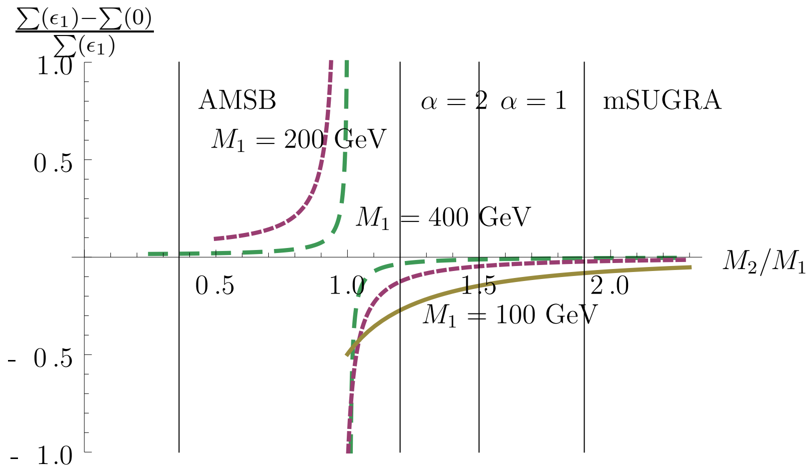


Figure 3.13: The fraction of the contribution arising from ϵ_1 to the total sum of (3.7.5) plotted as function of the ratio M_2/M_1 with $M_1 = 400$ GeV (long dashes, green), $M_1 = 200$ GeV (short dashes, purple), and $M_1 = 100$ GeV (solid line, ochre). On the horizontal axis $M_2/M_1 = 0.36$ corresponds to AMSB, $M_2/M_1 = 1.2$ to mirage mediation with $\alpha = 2$, $M_2/M_1 = 1.5$ to mirage mediation with $\alpha = 1$, and $M_2/M_1 = 1.9$ to mSUGRA. Here $\epsilon_1 = -0.1$ and $\tan\beta = 10$. Only the parts of the lines that agree with the experimental limit for the chargino mass (3.2.1) are shown.

Chapter 4

Renormalization group invariants and sum rules

4.1 Effective field theories

It stands to reason to expect that the MSSM (or other current model) is an approximate description of a more complete theory and insufficient at scales higher than a certain threshold energy. This new theory might in turn have its own limited region of applicability and be replaced by yet another theory at certain scale.

At specific energy scale, new fields could emerge and interact. These new fields must be explicitly included in the theory at higher energies, while the effects of the new fields are manifested through shifts in the masses of the remaining fields and couplings (as compared to masses and couplings in the new theory). The MSSM is then the low energy limit of the new theory and should follow from "integrating out" the new fields from the high energy theory. This approach is known as the effective field approach.

Let us assume that we have a quantum field theory with some fundamental scale M (e.g. mass of a heavy field). We then choose a cut-off $\Lambda < M$ up to which we expect a low-energy EFT to be accurate. The degrees of freedom heavier than Λ are removed by performing the integrals over these fields in the functional integral to give what is known as "Wilsonian effective action", which is non-local as a result of integrating out the heavier fields. This can be then expanded in terms of local operators using an "effective Lagrangian"

$$\mathcal{L} = \sum_i g_i Q_i \tag{4.1.1}$$

Table 4.1: Classification of operators and couplings in the effective Lagrangian [137]

Dimension	Importance for $E \to 0$	Terminology
$d_i < 4$	grows	relevant operators (super-renormalizable)
$d_i = 4$	constant	marginal operators (renormalizable)
$d_i > 4$	falls	irrelevant operators (non-renormalizable)

where the Q_i are operators consisting of fields with masses below an energy cut-off and the g_i are couplings containing information on high scale degrees of freedom. In general, all operators allowed by the symmetries of the theory are generated in the construction of the effective Lagrangian and appear in this sum. For more detailed explanation of the process of the derivation of effective Lagrangian see e.g. [137].

Since a Lagrangian has mass dimension of 4 we can determine using naive dimensional analysis:

$$[Q_i] \equiv d_i \qquad \Rightarrow \qquad g_i = \frac{C_i}{M^{d_i-4}} \tag{4.1.2}$$

where Λ is the energy cut-off below which the EFT is considered appropriate and C_i is a dimensionless constant, of $\mathcal{O}(1)$ due to hypothesis of naturalness, if we assume that M is the only fundamental scale in the theory.

At low energy ($E \ll \Lambda$), the contribution of a given operator Q_i in the effective Lagrangian to an observable (which for simplicity we assume to be dimensionless) is expected to scale as [137]

$$C_i \left(\frac{E}{M}\right)^{\gamma_i} = \begin{cases} O(1)\,; & \text{if } \gamma_i = 0, \\ \ll 1\,; & \text{if } \gamma_i > 0, \\ \gg 1\,; & \text{if } \gamma_i < 0, \end{cases} \tag{4.1.3}$$

where $\gamma_i = d_i - 4$. It follows that operators whose couplings have $\gamma_i \leq 0$ (or $d_i < 4$) are important at low energy. These operators are referred to as *relevant operators*. Operators whose couplings have $\gamma_i = 0$ contribute scale-independently and are referred to as *marginal operators*. Relevant and marginal operators largely determine the low-energy physics of the EFT. If $\gamma_i < 0$, the operator is called *irrelevant operator*. Its contribution grows with energy. Irrelevant or non-renormalizable operators can tell us about physics at the cut-off Λ. Once the series it terminated at give order of E/M (according to the desired precision), a finite number of operators are left in the effective Lagrangian. Non-renormalizable operators are allowed in an EFT since the theory is only expected to function to a cut-off at high energies and the operators are suppressed.

4.2 Probing the high scale structure

One of the basic quests for particle physics is the determination of how Nature functions at very high energies, in other words what are the properties of the theory valid at the unification or other basic defining scale. As we discussed in Chapter 2, a crucial question is how the supersymmetry is broken (assuming that the supersymmetry partners in fact exist). As the masses of the particles of a model at the scales accessible to collider measurements are determined by a small number of parameters at the high scale, usually the GUT-scale, one has to consider how the structure of the supersymmetry breaking can be deduced from TeV-scale measurements of the particle masses. Since the RGEs determining the evolution of the masses are not solvable analytically, all analysis must be performed numerically. Two common approaches discussed in the literature are referred to as the "top-down" and "bottom-up" methods.

4.2.1 Top-down method

As name suggests, the top-down method involves choosing a model and providing parameter values at the GUT-scale, then evolving the particle masses down to the TeV-scale. A detector simulation can be then performed to obtain verifiable predictions. The obvious limitation of this method is that predictions have to be produced separately for each point of the parameter space (and for each of the possible supersymmetry breaking scenarios) and compared to experimental data. The relationship of a theory with higher number of fields and its low energy approximation can be described using the effective field theory formalism.

4.2.2 Bottom-up method

An alternative approach, which is complementary to the “top–down” approach, that has been advocated involves the measurement of masses at the electroweak scale and evolving them to the high scale where supersymmetry is broken [138, 139, 140, 141]. The resulting structure is then analysed and conclusions about the underlying theory at high scales obtained. This approach, which can be called “bottom-up” approach, has uncertainties resulting from the present experimental uncertainties in the measurement of gauge and Yukawa couplings.

4.3 Renormalization group invariants

A method complementary to the two involves so called renormalization group invariants (RGIs) which are functions of the running parameters, composed in such way that they are invariant under renormalization group running at one-loop level. This property implies that any RGI measured at a collider-level energy scale has the same value at the GUT scale (or whatever the upper limit for which the theory remains valid is). With a sufficient number of measured low-scale parameters, it is possible to express a subset of all mass parameters as functions of RGIs. The knowledge of the high scale values of the masses can then provide information about the supersymmetry breaking structure.

RGIs have several important advantages over the other methods.

For instance, several RGIs are predicted to vanish in specific SUSY-breaking models, providing an instant ”diagnostics test” for excluding a class of models without information of the high scale parameters. Another attractive property is that in most SUSY breaking theories the fundamental high scale parameters can be expressed in terms of RGIs. RGIs may provide a way not only to exclude classes of theories, but also narrow the parameter range once a favoured class of theories is established from collider measurements.

Since none of the RGIs contains all the soft parameters and most only contain a few, RGIs may provide a way to test high scale properties with a more limited set of parameters than conventional approaches. RGIs simplify the bottom-up method since the integration of RGEs is avoided. It is also possible to derive renormalization scale independent sum rules from the RGIs which might provide an additional means for excluding models.

4.3.1 Renormalization group invariants of the MSSM

Complete one-loop renormalization group invariants for the MSSM together with corresponding sum rules have been derived in [142].

The derivation of RGIs is performed under several simplifying assumptions. They are derived from one-loop renormalization group equations, thus second order corrections are not taken into account. The magnitude of second order corrections is examined in [142] and concluded to be negligible or easily absorbed to a good approximation into a simple shift of the measured value of the RGIs. Also, the first and the second generation Yukawas are neglected as they give contributions to the evolution of the soft parameters that are smaller than the second order corrections to the third generation masses. Additionally it is assumed that there are no sources of CP-violation other than the Yukawa coefficients and that the right handed neutrino either does not exist or decouples from the spectrum. We will examine how the RGIs are modified in the presence of additional fields in the case that gauge mediated contributions to soft SUSY breaking.

As an illustrative example of how RGIs can be derived we examine a simpler case. By considering the renormalization group beta functions (with $\beta(p) \equiv 16\pi^2 \frac{dp}{dt}$ and $t = \log(\mu/\mu_0)$) for the gauginos and for the coupling constants ($a = 1, 2, 3$),

$$\begin{aligned} \beta(g_a) &= b_a g_a^3, && (4.3.1) \\ \beta(M_a) &= 2 b_a g_a^2 M_a, && (4.3.2) \end{aligned}$$

and by noting that $\beta(\frac{M_r}{g_r^2}) = 0$, we can define a quantity that is constant under renormalization group evolution,

$$I_{B_a} \equiv \frac{M_a}{g_a^2}. \qquad (4.3.3)$$

Thus we have derived three invariants, with six unknown parameters, and we are able to write

$$M_a(\mu) = I_{B_a} g_a^2(\mu). \qquad (4.3.4)$$

Thus I_{B_a} can be viewed as the constant of proportionality between M_a and g_a^2. If we were to measure a gaugino mass (and thus I_{B_r}) at a collider scale, we could then use the RGEs of the coupling constants to calculate the value of g_a at a high scale. Thus we have the value of M_a at any scale within the applicability of the effective field theory.

RGIs involving scalar masses can be derived in a similar fashion.

From the beta functions one can define in total twelve invariants which we have enumerated in Tables. 4.2 and 4.3. The invariants are linearly independent of each other. Naturally any linear combination of the RGIs is also an RGI, as is any function of the RGIs.

4.3.2 Renormalization group invariants in effective field theories and the deflected mirage mediation

By definition RGIs do not change their value within a theory that is characterised by specific beta functions. When the theory becomes inaccurate at some scale, and a new EFT with a particle content and beta functions that differ from the previous one must be applied, one must take care to examine what are the implications for the RGIs. This is the case when a gauge mediation component is introduced to the supersymmetry breaking mechanism, as is the case in e.g. the deflected mirage mediation and various other gauge mediation scenarios. We examine the case where N pairs of $SU(5)$ gauge messengers associated with a mass scale Λ appear at a messenger scale $\mu_{\rm mess}$. For simplicity we investigate how the gauginos and the associated RGIs I_{B_r} are modified when the threshold is crossed and the particle content is expanded. When the messengers are integrated out, the gaugino masses receive a correction [90]

$$M_r(\mu^-_{\rm MESS}) = M_r(\mu^+_{\rm MESS}) + \Delta M_r, \tag{4.3.5}$$

where $\mu^-_{\rm MESS}$ and $\mu^+_{\rm MESS}$ denote the renormalization scale just below and above $\mu_{\rm mess}$, respectively, and

$$\Delta M_r = -N\frac{g_a^2}{16\pi^2}\left(\Lambda + m_{3/2}\right), \tag{4.3.6}$$

where Λ is a mass scale associated with the messengers. Coupling constants, on the other hand, do not receive threshold corrections. Thus

$$g_r(\mu^-_{\rm MESS}) = g_r(\mu^+_{\rm MESS}) = g_r(\mu_{\rm mess}). \tag{4.3.7}$$

Just above the messenger scale the value of the invariant I_{B_r} is equal to the GUT-scale value,

$$I_{B_r}(\mu^+_{\rm mess}) \;\; = \tfrac{M_r(\mu^+_{\rm mess})}{g_r^2(\mu_{\rm mess})} = I_{B_r}(\mu_{\rm GUT}). \tag{4.3.8}$$

We now define evaluation at $\mu^+_{\rm mess}$ as evaluation at $\mu_{\rm mess}$ with modified coefficients b'_a and without the threshold corrections added to gaugino and scalar masses, and $\mu^-_{\rm mess}$ as evaluation at $\mu_{\rm mess}$ with the usual MSSM coefficients b_a and with threshold corrections added. Below the messenger scale the gaugino masses receive the threshold correction (2.2.29). Consequently just below the messenger scale

$$\begin{aligned} I_{B_r}(\mu^-_{\rm mess}) &= \tfrac{M_r(\mu^-_{\rm mess})}{g_r^2(\mu_{\rm mess})} = \tfrac{M_r(\mu^+_{\rm mess})}{g_r^2(\mu_{\rm mess})} + \tfrac{\Delta M_r}{g_r^2(\mu_{\rm mess})} \\ &= I_{B_r}(\mu_{\rm GUT}) + \Delta I_{B_r}, \end{aligned} \tag{4.3.9}$$

where ΔM_r is as in (2.2.29) and we have defined

$$\Delta I_{B_r} \equiv \Delta M_r / g_r^2(\mu_{\rm mess}) = -N/(16\pi^2)(\Lambda + m_{3/2}). \tag{4.3.10}$$

The couplings can be evolved down from the GUT scale to obtain

$$\begin{aligned} g_1(\mu_{\rm mess}) &= \frac{2\sqrt{10}\pi}{\sqrt{40\pi^2/g^2 + (33+5N)t_{\rm mess}}}, \\ g_2(\mu_{\rm mess}) &= \frac{2\sqrt{2}\pi}{\sqrt{8\pi^2/g^2 + (1+N)t_{\rm mess}}}, \\ g_3(\mu_{\rm mess}) &= \frac{2\sqrt{2}\pi}{\sqrt{8\pi^2/g^2 + (N-3)t_{\rm mess}}}, \end{aligned} \tag{4.3.11}$$

where $t_{\rm mess} = \ln \mu_{\rm GUT}/\mu_{\rm mess}$ and $g = g_1(\mu_{\rm GUT}) = g_2(\mu_{\rm GUT}) = g_3(\mu_{\rm GUT})$. Thus while the RGIs remain constant from messenger scale to GUT scale as well as from the electroweak scale to the messenger scale, there is a discontinuity at the messenger scale, which must be taken into account, unless the threshold contributions are cancelled out.

The invariants designated D_I are linear combinations of the squared scalar masses with the GUT-scale value of the from

$$D_I = \gamma_I m^2_{3/2} + \delta_I M_0^2. \tag{4.3.12}$$

The invariants are constructed in such a way that threshold corrections (2.2.30) cancel at the messenger scale, thus $D_I(\mu_{\rm TeV}) = D_I(\mu_{\rm GUT})$ and require no modifications from gauge mediation. We present D_{χ_1} as an example with the GUT-scale value

$$\begin{aligned} D_{\chi_1} &\equiv 3[3m^2_{\tilde{d}_1} - 2(m^2_{\tilde{Q}_1} - m^2_{\tilde{L}_1}) - m^2_{\tilde{u}_1}] - m^2_{\tilde{e}_1} \\ &= M_0^2(5 + 3n_U - 9n_D - 6n_L + n_E + 6n_Q). \end{aligned} \tag{4.3.13}$$

We use the scalar mass based invariants D_I to derive the high energy parameters of the deflected mirage mediation in terms of the scalar masses. The three invariants I_{M_a} are linear combinations of the squares of both scalar and gaugino masses, and are also explicitly dependent on b_a, e.g.

$$I_{M_1} = M_1^2 - \frac{5b_1}{8}(m^2_{\tilde{d}_1} - m^2_{\tilde{u}_1} - m^2_{\tilde{e}_1}), \tag{4.3.14}$$

where b_1 is replaced by $b_1' = b_1 + N$ above the messenger scale. Thus the shift at the messenger scale has a complex form,

$$\begin{aligned}
\Delta I_{M_1} &\equiv I_{M_1}(\mu^+_{\text{mess}}) - I_{M_1}(\mu^-_{\text{mess}}) \\
&= (M_1 + \Delta M_1)^2 - \frac{5b_1}{8}\Big((m^2_{\tilde{d}_1} + \Delta m^2_{\tilde{d}_1}) - (m^2_{\tilde{u}_1} + \Delta m^2_{\tilde{u}_1}) - (m^2_{\tilde{e}_1} + \Delta m^2_{\tilde{e}_1})\Big) \\
&- M_1^2 + \frac{5b_1'}{8}(m^2_{\tilde{d}_1} - m^2_{\tilde{u}_1} - m^2_{\tilde{e}_1}) \\
&= -2M_1\Delta M_1 - \Delta M_1^2 + \frac{5N}{8}(m^2_{\tilde{d}_1} - m^2_{\tilde{u}_1} - m^2_{\tilde{e}_1}) \\
&- \frac{5b_1}{8}(\Delta m^2_{\tilde{d}_1} - \Delta m^2_{\tilde{u}_1} - \Delta m^2_{\tilde{e}_1}),
\end{aligned} \tag{4.3.15}$$

where the gaugino mass and the scalar squareds are evaluated at μ^+_{mess} and ΔM_r and Δm_i are defined in (2.2.29) and (2.2.30). Since ΔI_{M_i} depends on masses at the messenger scale, accessing GUT-scale values from the TeV scale measurements is not as straightforward as with D_I. As with other invariants, ΔI_{M_r} is generated by the messengers and vanishes when the messengers are removed with $N = 0$.

We have listed the correction at the messenger scale and the value at the GUT-scale for each invariant in Table 4.2, except for the D_I invariants which are listed in Table 4.3. Fig. 4.1. shows the values of the invariants I_{B_a} and the square roots of I_{M_a}, and D_I above and below the messenger scale at the point $M_0 = 3$ TeV, $N = 3$, $\alpha_m = 1$, $\alpha_g = -0.5$, $t_{\text{MESS}} = -10$, $n_u = 1/2$, and $n_h = 1$.

4.4 Solving high scale parameters from the invariants

The MSSM contains 18 unknown soft mass parameters and gauge couplings. If we assume all RGIs to be non-zero, 14 of the parameters can be solved by

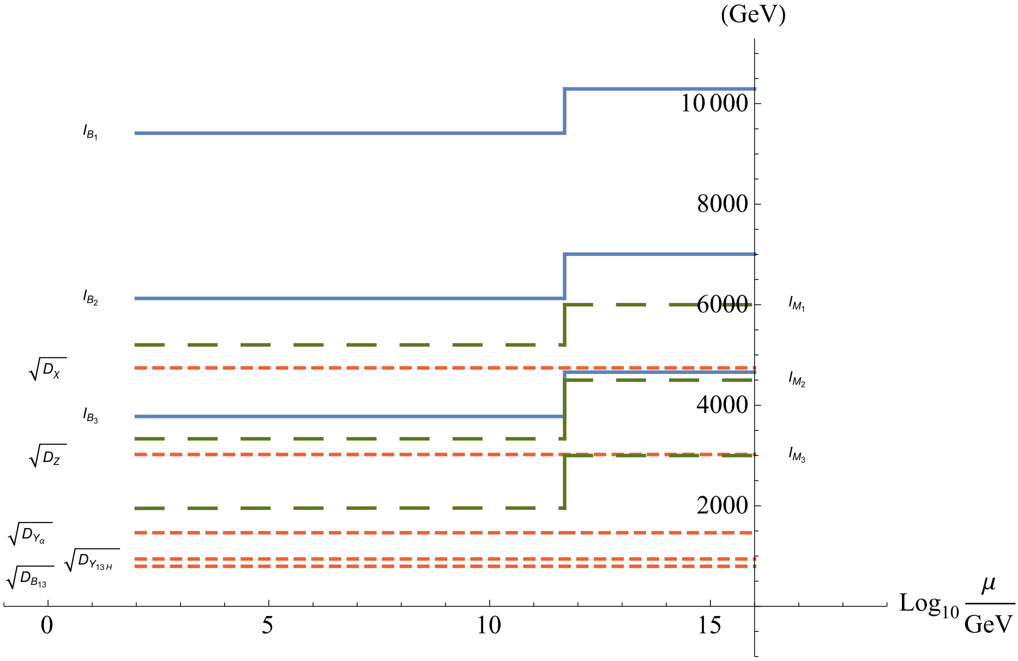


Figure 4.1: The renormalization group invariants I_{B_a} (blue, solid) and the square roots of the absolute value of the invariants D_I (red, small dash) and I_{M_a} (green, large dash). Here $n_u = 1/2$, $n_h =$, $M_0 = 3$ TeV $\alpha_m = 1$, $\alpha_g = -0.5$, $\mu_{\text{MESS}} = 10^{12}$ GeV, and $N = 3$. $D_{L_{13}} = 0$ is not shown.

Invariant	Definition	Correction at the messenger scale	Value at the GUT scale
I_{B_r}	M_r/g_r^2	$\Delta M_r/g_r^2$	$M_0/g^2 + \frac{b'_a}{16\pi^2} m_{3/2}$
I_{M_1}	$M_1^2 - \frac{5b_1}{8}(m^2_{\tilde{d}_1} - m^2_{\tilde{u}_1} - m^2_{\tilde{e}_1})$	$-2M_1\Delta M_1 - \Delta M_1^2 +$ $\frac{5N}{8}(m^2_{\tilde{d}_1} - m^2_{\tilde{u}_1} - m^2_{\tilde{e}_1})$ $-\frac{5b_1}{80}\Delta m^2 g_1^4$	$M_0^2(1 + n_{\epsilon_1} b'_1)$
I_{M_2}	$M_2^2 + \frac{b_2}{24}\left(9(m^2_{\tilde{d}_1} - m^2_{\tilde{u}_1}) + 16m^2_{\tilde{L}_1} - m^2_{\tilde{e}_1}\right)$	$-2M_2\Delta M_2 - \Delta M_2^2 -$ $\frac{N}{24}\left(9(m^2_{\tilde{d}_1} - m^2_{\tilde{u}_1}) + 16m^2_{\tilde{L}_1} - m^2_{\tilde{e}_1}\right)$ $+\frac{3b_2}{48}\Delta m^2 g_2^4$	$M_0^2(1 + n_{\epsilon_2} b'_2)$
I_{M_3}	$M_3^2 + \frac{b_3}{16}(5m^2_{\tilde{d}_1} + m^2_{\tilde{u}_1} - m^2_{\tilde{e}_1})$	$2M_3\Delta M_3 + \Delta M_3^2 -$ $\frac{N}{16}(5m^2_{\tilde{d}_1} + m^2_{\tilde{u}_1} - m^2_{\tilde{e}_1})$ $+\frac{b_3}{16}\Delta m^2 g_3^4$	$M_0^2(1 + n_{\epsilon_3} b'_3)$
I_{g_2}	$1/g_1^2 - (b_1/b_2)g_2^{-2}$	$28N/(5g_2^2(1+N))$	$1/g^2\,(1 - b'_1/b'_2)$
I_{g_3}	$1/g_1^2 - (b_1/b_3)g_3^{-2}$	$-16N/(5g_3^2(3-N))$	$1/g^2\,(1 - b'_1/b'_3)$

Table 4.2: The renormalization group invariants I_A involving scalar masses, gaugino masses and coupling constants. The second column defines the invariant in terms of soft masses and couplings without messenger fields present. The third column describes the difference of the value of the invariant above and below messenger scale; the masses and the couplings are to be evaluated at the messenger scale. The fourth column describes the value of the invariant at the GUT scale; the couplings are to be evaluated at the GUT scale. The quantity Δm^2 is defined as $N/(16\pi^4)\left(M_0\alpha_m(1+\alpha_g)\ln\frac{M_P}{m_{3/2}}\right)^2$.The combinations of modular weights n_{ϵ_a} are defined in [3]

measuring the invariants. However, if we restrict to a specific supersymmetry breaking scenario where the soft masses are determined by several input parameters at the input scale, the number of free parameters reduces. For example in the deflected mirage mediation scenario the spectrum is determined by M_0, $m_{3/2}$, Λ, N, and μ_{mess}. Also in the most general case the scalar masses include the 13 modular weighs n_i. If one restricts to a model in which are the modular weighs are identical or identical for all matter fields while different for the Higgses, the number of free parameters are greatly reduced. Some scenarios derived from string theories also assign specific values to n_i, leaving only the five remaining parameters free. Restricting to one or two of the contributing mediation mechanisms naturally constricts the parameters space further. RGIs provide a way to construct relations between the parameters, solve parameters, and determine modular weighs with measurement of low energy masses.

Following (4.3.9) we can write three equations involving the invariants I_{B_r} by setting the low energy scale value of I_{B_r} equal to the value at the GUT-scale corrected by the difference at the messenger scale,

$$I_{B_a}(\mu_{\mathrm{GUT}}) = I_{B_a}(\mu_{\mathrm{TeV}}) - \Delta I_{B_a}, \tag{4.4.1}$$

where I_{B_r} is defined in (4.3.3) and ΔI_{B_r} in (4.3.10). We note that ΔI_{B_r} vanishes if $\alpha_g = -1$. Thus the equivalence of the TeV-scale value of I_{B_a} to its GUT-scale value cannot be taken as proof of the absence of gauge messengers.

The equations (4.4.1) provide three independent solutions for $m_{3/2}$, and α_g, which we distinguish from each other by designating with the subindex (a),

$$m_{3/2\,(a)} = 16\pi^2 \frac{\sum_{b,c=1}^{3} \epsilon_{abc} I_{B_c}}{\sum_{d,e=1}^{3} \epsilon_{ade} b_e}, \qquad (a = 1, 2, 3), \tag{4.4.2}$$

$$\alpha_{g\,(a)} = \frac{b_a g^2 m_{3/2} - 16\pi^2 g^2 I_{B_a} + 16\pi^2 M_0}{g^2 m_{3/2} N}, \qquad (a = 1, 2, 3). \tag{4.4.3}$$

It is easy to verify by evolving g_i that the deflected mirage mediation coupling constant at the GUT scale, g, is related to g_{GUT}, which is the coupling constant at the GUT-scale with $N = 0$, by

$$\frac{1}{g} = \sqrt{\frac{1}{g_{\mathrm{GUT}}^2} - \frac{N t_{\mathrm{mess}}}{8\pi^2}}. \tag{4.4.4}$$

A different set of parameters to eliminate could, of course, be chosen, but this choice proves to be most convenient for solving all parameters, as M_0 is readily

I	Definition	γ_I	δ_I
D_Z	$3(m^2_{\tilde{d}_3} - m^2_{\tilde{d}_1}) + 2(m^2_{\tilde{L}_3} - m^2_{H_d})$	$Y_{Za} + Y_{Zb}g^2$	$2n_\alpha$
D_{χ_1}	$3(3m^2_{\tilde{d}_1} - 2(m^2_{\tilde{Q}_1} - m^2_{\tilde{L}_1}) - m^2_{\tilde{u}_1}) - m^2_{\tilde{e}_1}$	0	n_β
$D_{L_{13}}$	$2(m^2_{\tilde{L}_1} - m^2_{\tilde{L}_3}) - m^2_{\tilde{e}_1} + m^2_{\tilde{e}_3}$	0	0
$D_{B_{13}}$	$2(m^2_{\tilde{Q}_1} - m^2_{\tilde{Q}_3}) - m^2_{\tilde{u}_1} + m^2_{\tilde{u}_3} - m^2_{\tilde{d}_1} + m^2_{\tilde{d}_3}$	$Y_{B_{13}a} + Y_{B_{13}b}g^2$	0
$D_{Y_{13H}}$	$m^2_{\tilde{Q}_1} - 2m^2_{\tilde{u}_1} + m^2_{\tilde{d}_1} - m^2_{\tilde{L}_1} + m^2_{\tilde{e}_1}$ $-\frac{10}{13}\left(m^2_{\tilde{Q}_3} - 2m^2_{\tilde{u}_3} + m^2_{\tilde{d}_3} - m^2_{\tilde{L}_3} + m^2_{\tilde{e}_3} + m^2_{H_u} - m^2_{H_d}\right)$	$\frac{10}{13}\left(-Y_{\alpha 1} - Y_{\alpha 2}g^2\right)$	$-\frac{1}{13}n_\gamma$
$D_Y\alpha$	$\left(m^2_{H_u} - m^2_{H_d} + \sum_{gen}(m^2_{\tilde{Q}} - 2m^2_{\tilde{u}} + m^2_{\tilde{d}} - m^2_{\tilde{L}} + m^2_{\tilde{e}})\right)/g^2$	$\frac{1}{g^2}\left(Y_{\alpha 1} + Y_{\alpha 2}\right)$	$-\frac{1}{g^2}n_\delta$

Table 4.3: The invariants D_I and their GUT-scale values parametrised as $D_I(\mu_{\rm GUT}) = \gamma_I m^2_{3/2} + \delta_I M_0^2$. The combinations of modular weights n_A and the functions of Yukawa coefficients $Y_{\alpha k}$ are defined in [3]

solved from the scalar mass involving invariants D_I which do not allow the determination of α_g.

In similar fashion, all five high scale parameters for the deflected mirage mediation can be solved analytically using the invariants. We have examined the solutions in detail in [3].

4.5 Sum rules

One way to utilise RGIs is to construct sum rules that are valid regardless of the renormalization scale in order to test various properties of the theory. In the absence of the explicit knowledge of the RGIs at the input scale, one can eliminate some parameters from the RGIs to form sum rules. For example large class of theories involves gauge coupling unification at a high scale. Gaugino mass unification and scalar mass unification are two simple properties that can be tested using the sum rules constructed of the RGIs.

As a generic example we assume gaugino mass unification at some scale and write $M_1 = M_2 = M_3 = M_{1/2}$. From (4.3.3),

$$I_{B_a} = \frac{M_{1/2}}{g_a^2}. \tag{4.5.1}$$

By combining this to the definitions of the invariants I_{g_2} and I_{g_3},

$$\begin{aligned} I_{g_2} &= 1/g_1^2 - (b_1/b_2)g_2^{-2}, & (4.5.2)\\ I_{g_3} &= 1/g_1^2 - (b_1/b_3)g_3^{-2}, & (4.5.3) \end{aligned}$$

we can eliminate $M_{1/2}$ from the resulting group of equations to obtain the sum rule

$$[I_{B_1} - (b_1/b_3)I_{B_3}]I_{g_2} = [I_{B_1} - (b_1/b_2)I_{B_2}]I_{g_3}. \tag{4.5.4}$$

If we assume the gaugino mass unification to occur in conjunction with a scalar mass unification (with the common scalar mass squared value m_0^2) at the same scale, the RGIs I_{M_a} have the values

$$I_{M_1} = \frac{33m_0^2}{8} + M_{1/2}^2, \tag{4.5.5}$$

$$I_{M_2} = \frac{5m_0^2}{8} + M_{1/2}^2, \tag{4.5.6}$$

$$I_{M_3} = M_{1/2}^2 - \frac{15m_0^2}{16}. \tag{4.5.7}$$

This allows us to write the sum rule

$$81 I_{M_2} - 56 I_{M_3} - 25 I_{M_1} = 0. \tag{4.5.8}$$

Similarly from the assumption of gauge coupling unification at the GUT scale one can derive

$$I_{g_1} - I_{g_2}(1 - b_1/b_2)/(1 - b_1/b_3) = 0. \tag{4.5.9}$$

These and several other sum rules have been derived in [143, 144]. When the boundary conditions for the masses and the couplings for the theory are known, number of sum rules can be constructed by expressing the RGIs at the input scale using the boundary conditions and then eliminating the supersymmetry breaking parameters in order to construct sum rules. In the case of deflected mirage mediation we look several possible sum rules derived this way in [3].

4.6 Comparison of the RGIs in different supersymmetry breaking models

In order to examine contributions of different supersymmetry breaking mechanisms and their implications for the values of RGIs, we employ the deflected mirage mediation boundary conditions that include contributions from three separate supersymmetry breaking mechanisms, namely gravity mediation (SUGRA) [145, 49, 146, 147, 148], gauge mediation (GMSB) [54, 55, 56], and anomaly mediation (AMSB) [149, 60]. We can then examine the individual mechanism with various limits of the deflected mirage mediation parameters. The boundary conditions for the scalar and the gaugino masses for the three constituting mechanisms can be parametrised as

$$\text{SUGRA:} \quad m_i^2(\mu_{\mathrm{GUT}}) = (1 - n_i) M_0^2; \qquad M_a(\mu_{\mathrm{GUT}}) = M_0, \tag{4.6.1}$$

$$\text{AMSB:} \quad m_i^2(\mu_{\mathrm{GUT}}) = -\frac{\dot{\gamma}_i'}{(16\pi^2)^2} m_{3/2}^2; \quad M(\mu_{\mathrm{GUT}}) = g^2 \frac{b_a}{16\pi^2} m_{3/2}, \tag{4.6.2}$$

$$\text{GMSB:} \quad m_i^2(\mu_{\mathrm{MESS}}) = \frac{N\Lambda^2}{(16\pi^2)^2} \sum_{a=1}^{3} c_a(\Psi_i) g_a^4; \; M(\mu_{\mathrm{MESS}}) = \frac{N g_a^2}{16\pi^2} \Lambda. \tag{4.6.3}$$

Note that the GMSB boundary conditions are defined on the messenger scale possibly different from the GUT scale while the AMSB and SUGRA boundary conditions are defined at the GUT-scale. Additionally, two combinations of the above exist: mirage mediation (MMSB) is obtained from DMMSB by removing

Sum rule	$\mu < \mu_{\rm mess}$	$\mu > \mu_{\rm mess}$
$I_{g_1} - I_{g_2}(1-b_1/b_2)/(1-b_1/b_3) = 0$	OK	OK
$(I_{B_1} - (b_1/b_3)I_{B_3})I_{g_2} = (I_{B_1} - (b_1/b_2)I_{B_2})I_{g_3}$	OK	OK
$I_{g_2} = \left(I_{M_1} - \frac{b_1}{8}D_{\chi 1}\right)^{-1/2} I_{B_1} - \frac{b_1}{b_2}\left(I_{M_2} - \frac{b_2}{8}D_{\chi 1}\right)^{-1/2} I_{B_2}$	X	$n_i = n_u$
$I_{g_3} = \left(I_{M_1} - \frac{b_1}{8}D_{\chi 1}\right)^{-1/2} I_{B_1} + \frac{b_1}{b_3}\left(I_{M_3} + \frac{b_3}{16}D_{\chi 1}\right)^{-1/2} I_{B_3}$	X	$n_i = n_u$
$I_{M_1} - (2b_1+b_3)/(2b_2+b_3)I_{M_2} + 2(b_1-b_2)/(2b_2+b_3)I_{M_3} = 0$	X	$n_i = 1$

Table 4.4: Sum rules derived from the condition of gauge coupling unification, gaugino mass unification and scalar mass unification. Third and fourth rows describe whether the sum rule is valid above and below messenger scale respectively. The bottom three sum rules involving scalar masses are valid only above messenger scale and with the condition of universal modular weights $n_Q = n_U = n_D = n_E = n_L = n_u$. Above the messenger scale b'_a is to be substituted for b_a.

the messengers and deflected anomaly mediation (DAMSB) [61, 151, 152] is obtained by setting M_0 to zero.

The boundary conditions for the three models and the pure mirage mediation which combines AMSB and SUGRA can be extracted from the deflected mirage mediation boundary conditions for the gauginos and the scalars (2.2.26), (2.2.27), (2.2.30), and (2.2.29) with the following prescriptions:

$$\text{SUGRA:} \quad m_{3/2} = 0; \; N = 0, \tag{4.6.4}$$

$$\text{AMSB:} \quad M_0 = 0; \; N = 0, \tag{4.6.5}$$

$$\text{GMSB:} \quad M_0 = 0; \; m_{3/2} = 0, \tag{4.6.6}$$

$$\text{Pure mirage:} \quad N = 0, \tag{4.6.7}$$

$$\text{DAMSB:} \quad M_0 = 0. \tag{4.6.8}$$

Here we assume universal modular weights for all applicable models and the parameters not specified to be nonzero. In the case that one of the mechanisms clearly dominates supersymmetry breaking scenario can then in principle be resolved or narrowed by measuring the parameters in terms of RGIs. By comparing to the solutions for the parameters in terms of invariants, along with the sum rules derived for the deflected mirage mediation, we can deduce from the measured invariants I_{B_1}, I_{B_3}, I_{M_a}, $D_{Y\alpha}$, and $D_{Y_{13H}}$ the following:

$$I_{B_1} - I_{B_3} \propto m_{3/2} \begin{cases} 0, \text{ for mSUGRA, GMSB} \\ > 0, \text{ for AMSB, Mirage, DAMSB} \end{cases},$$

$$D_{\chi 1} = (1 - n_u) M_0^2 \begin{cases} = 0, \text{ for AMSB, GMSB} \\ \propto (1 - n_u), \text{ for mSUGRA, Pure Mirage, DAMSB} \end{cases},$$

$$\frac{10 D_{Y\alpha}}{13 D_{Y_{13H}}} + \frac{1}{g_{\text{GUT}}^2} = \frac{1}{g_{\text{GUT}}^2} - \frac{1}{g^2} \begin{cases} = 0 \text{ for mSUGRA, AMSB, Pure Mirage} \\ \propto t_{\text{MESS}} N, \text{ for GMSB, DAMSB} \end{cases},$$

$$81 I_{M_2} - 56 I_{M_3} - 25 I_{M_1} \begin{cases} = 0, \quad \text{for AMSB, mSUGRA, Pure Mirage} \\ \neq 0, \quad \text{for Deflected Mirage, GMSB, DAMSB} \end{cases}.$$

Thus e.g. observing $D_{\chi_1} = 0$ would exclude mediation mechanism with a gravity mediated contribution with $n_u \neq 1$, but deflected or pure mirage with $n_u = 1$ cannot be ruled out. On the other hand a nonzero $\frac{10 D_{Y\alpha}}{13 D_{Y_{13H}}} + \frac{1}{g_{\text{GUT}}^2}$ implicates a gauge mediated contribution, with the messenger scale different from the GUT scale. We have illustrated the implications of different values for the invariants in Fig. 4.2 by starting from the measurement of D_{χ_1}.

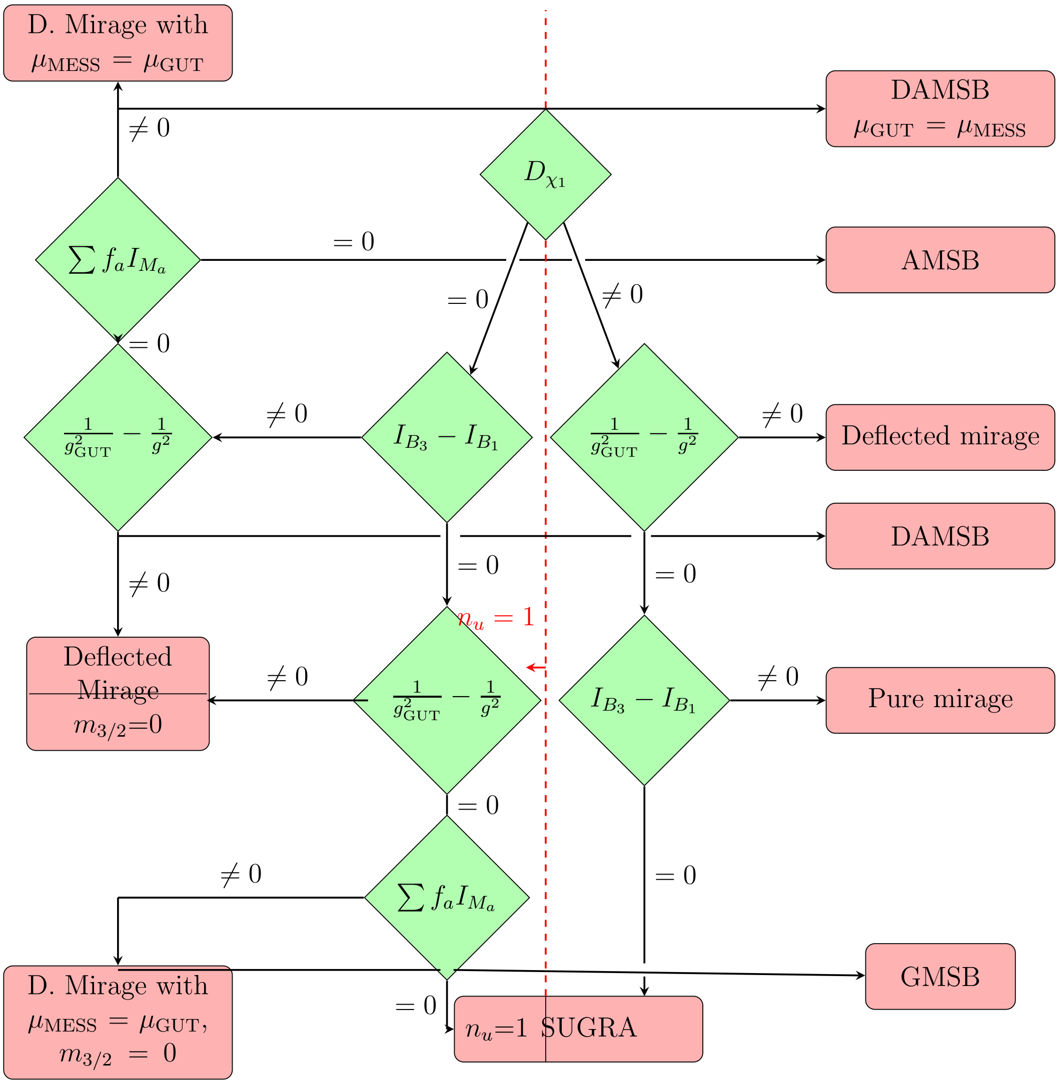


Figure 4.2: Implications of measurement of RGIs for the supersymmetry breaking mechanism with the assumption of $n_h = n_{H_d} = n_{H_u}$ and $n_u = n_U = n_D = n_L = n_E = n_Q$, assuming that one or two of the mechanisms dominate. On the left side of the red dotted line all endpoints have $n_u = 1$. $\frac{1}{g^2} = -\frac{10 D_{Y\alpha}}{13 D_{Y_{13H}}}$ and $\sum f_a I_{M_a} = 81 I_{M_2} - 56 I_{M_3} - 25 I_{M_1}$.

Chapter 5

Summary and future perspectives

The lightest neutralino is expected to be the lightest SUSY particle in SUSY models with R-parity conservation. It is expected to be the end product of decays of SUSY partners of the Standard Model particles. Thus, it's mass, and its properties are of considerable importance for the supersymmetric phenomenology. The mass of the lightest neutralino, as well as those of its heavier partners, depend heavily on the mechanism of supersymmetry breaking in the gaugino sector.

In [1] we have carried out a detailed study of the spectrum of neutralinos and charginos in three popular models of SUSY breaking by investigating the patterns of gaugino masses peculiar to each model and their implications, particularly for the properties of lightest neutralino.

By utilizing the gaugino mass patterns, we have derived lower limits on the masses of the neutralinos and the charginos based on the current experimental limits on the mass of the lightest chargino. Although these limits depend, through radiative corrections, on parameters other than those related to the gaugino sector, we have found that this dependence is mild, and thus the limits for the neutralino and chargino masses can be considered to be relatively robust.

We have also calculated an upper bound on the mass of the lightest neutralino as a function of the lightest chargino mass. We see that for the models of supersymmetry breaking considered in [1], only in the mirage mediation model with large α, the upper bound found from the lower right hand two-by-two part of the mass matrix becomes relevant.

The sum involving the difference of squared masses of neutralinos and charginos provides an additional route to distinguishing between the SUSY breaking mechanisms and their related gaugino mass patterns. In particular AMSB scenario produces negative sum as opposed to the positive sign produced by mSUGRA and mirage mediation. The relation of provided by the sum rule could aid in determining the value of α or the anomaly mediation to gauge mediation ratio.

We have also discussed in detail the decay patterns of the neutralinos and charginos in the three SUSY breaking models. A notable conclusion of our work is that detection of neutralino and chargino decay patterns gives important information on the nature of the underlying SUSY breaking mechanism, and may help in identifying the correct SUSY breaking pattern.

We have shown that the second lightest neutralino and the lighter chargino are produced in large amounts in squark decays. This is of importance, since a promising signal to detect weakly interacting particles at Tevatron and at LHC is considered to be the associated production $\tilde{\chi}_1^{\pm}\tilde{\chi}_2^0$, see *e.g.* [118, 119] and references therein. In the studied cascade decay production of $\tilde{\chi}_1^{\pm}$, $\tilde{\chi}_2^0$ we have observed that the magnitude of the trilepton signal varies significantly between the gaugino patterns.

In the mSUGRA pattern, $\tilde{t}_1$ decays to all the heavier neutralinos and charginos with non-negligible branching fractions. The contribution $\tilde{t}_1 \to \tilde{\chi}_1^+ b/\tilde{\chi}_2^0 t$ is at a few percent level, but more events come from the decays of $\tilde{\chi}_{3,4}^0$, $\tilde{\chi}_2^+$. Thus from $\tilde{t}_1\bar{\tilde{t}}_1$ production there is an additional contribution to the trilepton signal, accompanied by a number of jets. In the AMSB pattern, the enhancement of trileptons is significant. $\tilde{t}_1$'s decay 60% of the time to $\chi_2^0 t$ and 20 % of the time to $\chi_1^+ b$. As soon as kinematically possible, the χ_2^0 decays to a slepton and lepton, and χ_1^+ decays leptonically 25% of the time. In the mirage pattern, stops tend to decay directly to the lightest neutralino and no enhancement is expected.

Since the lightest neutralino is a possible candidate for particle dark matter, we have calculated its relic density in different SUSY breaking models combining the information coming from decay patterns. While in the mSUGRA model typically a narrow range with the observed relic density occurs, in the AMSB model the relic density remains below the WMAP limit for the sub-TeV scale spectrum. In mirage mediation models the observed dark matter range is narrow and close to the stop LSP region, unless the heavy Higgs resonance can be found. We note that it is not necessary that neutralino is the only dark matter particle, even if it were the lightest supersymmetric particle. Furthermore, it is possible that the R-parity is broken at least slightly in nature. This would lead to the neutralino decay, even if the breaking were so tiny that it would not show up in the experiments.

In [2] we investigated the contribution of a dimension five BMSSM operator to the neutralino and chargino masses in various SUSY breaking models. We observed that the contribution can be significant when the higgsino mixing parameter is small compared to the soft supersymmetry breaking gaugino mass parameters. If the parameter is large, its effect is negligible on the mass of the lightest neutralino, which is dominantly a gaugino. Thus the sensitivity to the BMSSM operator is very different in different supersymmetry breaking models, since in the mSUGRA and mirage mediation models the parameter can be small, while in the anomaly mediation models it is always larger than the gaugino mass parameters.

Renormalization group invariants are quantities, composed of mass parameters and couplings constants, that remain invariant under renormalization group running. They can be a valuable tool for determining high scale structure of the theory and thus determining the type of supersymmetry breaking involved. In [3] we investigated the behavior of RGIs in the deflected mirage mediation (DMMSB), which is the most general type of mechanism for spontaneous supersymmetry breaking in the sense that it includes contributions from three supersymmetry breaking mechanisms, namely gravity-, anomaly-, and gauge mediation.

In the case of DMMSB, the emergence of gauge messenger fields at a scale possibly different from the GUT-scale complicates the use of RGIS by inducing corrections to the gaugino and the scalar masses and modifying the beta functions at this threshold. Thus the RGIs have differing values above and below the messenger scale. In order to connect the TeV scale measurements of the particle masses to the GUT-scale parameters we have derived the threshold corrections to the RGIs and derived the RGIs for arbitrary b_a-coefficients of the beta functions.

It is shown that the high scale parameters which include N, $\mu_{\rm MESS}$, $m_{3/2}$, M_0, and α_g can be analytically solved in terms of the RGIs, and the explicit formulas are provided.

We have examined various limits of DMMSB to see how any of the contributing three pure supersymmetry breaking scenarios are manifested in the values of the RGIs at the TeV scale.

Sum rules formed from the RGIs can provide useful diagnostics test for excluding theories that are inconsistent with experiments. We have discussed how the solutions to the supersymmetry breaking parameters can be used to construct sum rules that would allow further testing of the theory and determine the modular weights for the scalar masses.

At the moment the existence of SUSY as a part of nature remains hypothetical, since the LHC or other empirical data is yet to produce evidence of the particles that would indicate that the MSSM or similar model provides an accurate description of our reality. For a low-mass LSP, the lower limit for the gluino mass has been pushed to 2-3 TeV for most models, ruling out parts of the the parameter ranges explored in [1] and [2]. At the moment we are still waiting for experimental confirmation of SUSY and data on the masses of the MSSM fields to be able to make inferences about the intricacies of the SUSY breaking mechanism and other properties of physics beyond the Standard Model.

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